

Closed timelike curves via post-selection: theory and experimental demonstration

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Closed timelike curves (CTCs) are trajectories in spacetime that effectively travel backwards in time: a test particle following a CTC can in principle interact with its former self in the past. CTCs appear in many solutions of Einstein’s field equations and any future quantum version of general relativity will have to reconcile them with the requirements of quantum mechanics and of quantum field theory. A widely accepted quantum theory of CTCs was proposed by Deutsch. Here we explore an alternative quantum formulation of CTCs and show that it is physically inequivalent to Deutsch’s. Because it is based on combining quantum teleportation with post-selection, the predictions/retrodictions of our theory are experimentally testable: we report the results of an experiment demonstrating our theory’s resolution of the well-known ‘grandfather paradox.’

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Although time travel is usually taken to be the stuff of science fiction, it is not ruled out by scientific fact. Einstein’s theory of general relativity admits the possibility of closed timelike curves (CTCs) [1], paths through spacetime which, if followed, allow a time traveller to go back in time and interact with her own past. The logical paradoxes inherent in time travel make it hard to formulate self-consistent physical theories of time travel [2–6]. This paper proposes an empirical self-consistency condition for closed timelike curves: we demand that a generalized measurement made before a quantum system enters a closed timelike curve yield the same statistics – including correlations with other measurements – as would result if the same measurement were made after the system exits from the curve. That is, the closed timelike curve behaves like an ideal, noiseless quantum channel that displaces systems in time without affecting the correlations with external systems. To satisfy this criterion without introducing contradictions, we construct a theory of closed timelike curves via quantum post-selection (P-CTCs). The theory is based on Bennett and Schumacher’s suggestion [7] to describe time travel in terms of quantum teleportation, and on the Horowitz-Maldacena model for black hole evaporation [8]. We show that P-CTCs are consistent with path integral approaches [9, 10], but physically inequivalent to the prevailing theory of closed timelike curves due to Deutsch [2]. Moreover, because they are based on post-selection [11], closed timelike curves can be simulated experimentally. We present an experimental realization of the grandfather paradox: the experiment tests what happens when a photon is sent a few billionths of a second back in time to try to ‘kill’ its former self.

Deutsch’s elegant quantum treatment of closed timelike curves [2] provides a self-consistent resolution of the

various paradoxes of time travel by requiring simply that a system that enters such a curve in a particular quantum state ρ , emerges in the past in the same state (Fig. 1a) even after interacting with a “chronology-respecting” system in a state ρ_A through a unitary U . This translates into the consistency condition,

$$\rho = \text{Tr}_A[U(\rho \otimes \rho_A)U^\dagger]. \quad (1)$$

A state ρ that satisfies Eq. (1) always exists because the above interaction is a completely positive map which possesses at least one fixed point.

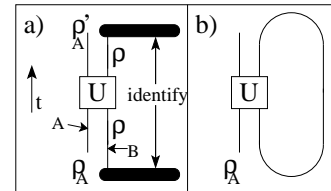


FIG. 1: a) Deutsch’s quantum description of CTCs is based on the consistency condition of Eq. (1), where the unitary U describes an interaction between a chronology-respecting system A , initially in the state ρ_A , and a system B in a CTC. Deutsch demands that the state ρ of B at the input and output of U be equal, inducing a nonlinear transformation $\rho_A \rightarrow \rho'_A$. Time goes from bottom to top in this and in the following diagrams. b) P-CTC: post-selected quantum teleportation is employed as a description of the closed timelike curve. The bottom curve \cup represents the creation of a maximally entangled state of two systems and the upper curve \cap represents the projection onto the same state.

Here we propose an alternative consistency condition for CTCs: a generalized measurement made on the state entering the curve should yield the same results, including correlations with other measurements, as would occur if the same measurement were made on the state

emerging from the curve. The CTC should behave like an ideal quantum channel (even though, as we shall see, inside a CTC a proper definition of state cannot be given). Deutsch’s CTCs fail this requirement, thus are physically inequivalent to our proposed solution. To make the CTC behave like a quantum channel, we describe it using quantum teleportation [12], namely the perfect transfer of an unknown quantum state $|\psi\rangle$ between two parties (Alice and Bob) using a shared entangled state, the transmission V on Bob’s side. A curious feature of teleportation is that, whenever Alice’s Bell measurement gives the same result it would when measuring the initial shared state, then Bob’s unitary V is the identity. In this case, Bob does not have to perform any transformation to obtain Alice’s state $|\psi\rangle$: in some sense, Bob possesses the unknown state even before Alice implements the teleportation! Causality is not violated because Bob cannot foresee Alice’s measurement result, which is completely random. However, if we could pick out only the proper result with probability one, the resulting ‘projective’ teleportation would allow information to propagate along spacelike intervals, to escape from black holes [8], or to travel backwards in time along a closed timelike curve. We call this mechanism a projective or post-selected CTC, or P-CTC.

The P-CTC (see Fig. 1b) starts from two systems prepared in a maximally entangled state “ \cup ”, and ends by projecting them into the same state “ \cap ”. If non-zero, the probability amplitude of the final state is renormalized to one, a nonlinear process. If zero, the event cannot happen: our mechanism embodies in a natural way the Novikov principle [13] that only *logically self-consistent* sequences of events occur in the universe. Because they rely on post-selection, P-CTCs share some properties with the weak value interpretation of quantum mechanics [14], notably that there is no unique way to assign a definite state to the system in a CTC at a definite time. This is not surprising due to the cyclic nature of time there. Moreover, Hartle [10] showed that quantum mechanics on closed timelike curves is non-unitary (indeed, it allows cloning) and requires events in the future to affect the past. He noted that the Hilbert space formalism for quantum mechanics might be inadequate to capture the behavior of closed timelike curves, and suggested a path integral approach instead. In future work we will show that, in contrast [9] to Deutsch’s, P-CTCs are consistent with the “traditional” path-integral approaches to CTCs (e.g. see [5, 9, 10, 15]): we use the normal path-integral self-consistency requirement that the classical paths that make up the path integral have the same values of all variables (e.g. x and p) when they exit the CTC as when they enter. For example, our approach coincides with Politzer’s [9] path-integral treatment of fermions. However, P-CTCs can also be described in Hilbert space, showing that this approach can be reconciled with path integrals when post-selection is allowed.

We now analyze how P-CTC deal with time travel paradoxes. In the grandfather paradox, for example, the time traveller goes back in time and kills her grandfather, so she cannot be born and cannot kill her grandfather: a logical contradiction. This paradox can be implemented through a quantum circuit where a ‘living’ qubit (i.e., a bit in the state 1), goes back in time and tries to ‘kill’ itself, i.e., flip to the state 0, see Fig. 2a. There are many possible variants: but any circuit in which travels back in time and gives rise to intrinsic logical self-contradiction is an embodiment of the grandfather paradox. Deutsch’s consistency condition (1) requires that the state is $\rho = (|0\rangle\langle 0| + |1\rangle\langle 1|)/2$, the only fixed point of the corresponding map. Note that if the CNOT before the bit flip measures a 0 then the CNOT afterwards measures a 1, and *vice versa*: the time traveller really manages to kill her grandfather! So far, so good. The strange aspect of Deutsch’s solution comes when one attempts to follow the state of the time-traveller through the CTC. To preserve self-consistency, the 1 component (time traveller alive) that enters the loop emerges as the 0 component (time traveller dead), while the 0 component (time traveller dead) that enters the loop emerges as the 1 component (time traveller alive). Thus, the CTC preserves the overall mixed state, but not the identity of the components: projective measurements at the input and output yield different results.

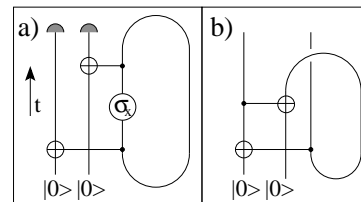


FIG. 2: a) Grandfather paradox circuit. If we take 1 to represent ‘time-traveler exists,’ and 0 to represent ‘she doesn’t exist,’ then the NOT (σ_x) operation implies that if she exists, then she ‘kills her grandfather’ and ceases to exist; conversely, if she doesn’t exist, then she fails to kill her grandfather and so she exists. The difference between Deutsch’s CTCs and our P-CTCs is revealed by monitoring the time-traveler with two controlled-NOTs (CNOT): the two controlled bits are measured to determine the value of the time-traveling bit before and after the σ_x . Opposite values mean she has killed her grandfather; same values mean she has failed. Using Deutsch’s CTCs, she always succeeds; using P-CTCs she always fails. b) Unproved theorem paradox circuit. The time-traveler obtains a bit of information from the future via the upper CNOT. She then takes it back in time and deposits a copy an earlier time in the same location from which she obtained it (rather, will obtain it), via the lower CNOT. Because the circuit is unbiased as to the value of the ‘proof’ bit, it automatically assigns that bit a completely mixed value, since it is maximally entangled with the one emerging from the post-selected CTC.

P-CTCs give a different resolution of the grandfather

paradox: the probability amplitude of the projection onto the final entangled state \cap is always null, namely this event (and all logically contradictory ones) cannot happen. In any real-world situations, the σ_x transformation is not perfect. Then, replacing σ_x with $e^{-i\theta\sigma_x} = \cos\frac{\theta}{2}\mathbb{1} - i\sin\frac{\theta}{2}\sigma_x$ (with $\theta \simeq \pi$), the non-linear post-selection amplifies fluctuations of θ away from π . This eliminates the histories plagued by the paradox and retains only the self-consistent histories in which the time-traveler fails to kill her grandfather (the unitary in the curve is $\mathbb{1}$ instead of σ_x), and the two output qubits have equal value: P-CTC fulfill our self-consistency condition. In other words, no matter how hard the time-traveler tries, she finds her grandfather a tough guy to kill.

Because P-CTCs are based on post-selected teleportation, their predictions can be experimentally demonstrated. To experimentally demonstrate the grandfather paradox, we store two qubits in a single photon: one in the polarization degree of freedom, which represents the forward-travelling qubit, and one in a path degree of freedom representing the backward travelling qubit as shown in Fig 3. Our single photons, with a wavelength of 941.7 nm, are coupled into a single-mode fiber from an InGaAs/GaAs quantum dot cooled to 21.5K by liquid Helium [16] and sent to the circuit. Using a Hanbury-Brown-Twiss interferometer, the $g^{(2)}(0)$ of the quantum dot emission was measured to be 0.29 ± 0.01 , confirming the single-photon character of the source. At the start of the circuit, (\cup) we entangle the path and polarization qubits using a beam displacer (BD1), generating the Bell state $|\phi^+\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$. To close the CTC (\cap), we perform a Bell state measurement and post-select on cases where $|\phi^+\rangle$ is detected. The Bell state measurement consists of a CNOT with polarization (forward traveller) acting on path (backward traveller), followed by post-selection on the now-disentangled qubits. The CNOT is implemented by a polarizing beam splitter that flips the backward-travelling (path) qubit conditioned on the value of the forward-travelling (polarization) qubit. We then post-select on photons exiting the appropriate spatial port using a polarizer at 45° and an Andor iDus CCD camera cooled to 188K. Within the loop, we implement a “quantum gun” $e^{i\theta\sigma_x}$ with a wave plate that rotates the polarization by an angle $\theta/2$. The accuracy of the quantum gun can be varied from $\theta = \pi$ (the photon “kills” its past self) to $\theta = 0$ (the photon always “misses” and survives).

The teleportation circuit forms a polarization interferometer whose visibility was measured to be $93 \pm 3\%$ (see the inset in Fig. 4). To verify the operation of the teleportation circuit, all four Bell states $|\phi^\pm\rangle, |\psi^\pm\rangle$ were prepared and sent to the measurement apparatus: post-selection on $|\phi^+\rangle$ behaved as expected yielding success probabilities of 0.96 ± 0.08 , 0.10 ± 0.11 , 0.02 ± 0.05 , and 0.02 ± 0.05 for $|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle$, and $|\psi^-\rangle$ inputs respectively. After verifying the operation of the teleporta-

tion circuit, calcite beam displacers were inserted (BD2 and BD3), coupling the polarization qubit to two probe qubits encoded in additional path degrees of freedom of the photon. These probe qubits measure the state of the polarization qubit before and after the quantum gun is “fired”. When the post-selection succeeds (i.e. the time travel occurs), the state of the probe qubits is measured. If the two probe qubits are in agreement (00 or 11) the quantum gun has failed to flip the polarization and the photon “survives”. If the two probe qubits disagree, the photon has “killed” its past self.

The state of the probe qubits, conditioned on the time travel succeeding, was measured for different values of θ , see Fig. 4. The probe qubits are never found in the states 01 or 10: time travel succeeds only when the quantum gun misfires, leaving the polarization unchanged and the probe qubits in 00 or 11. Our suicidal photons obey the Novikov principle and never succeed in travelling back in time and killing their former selves. The required non-linearity is due to post-selection here: no CTCs nor any evidence of the nonlinear signature of a P-CTC has ever been observed in nature up to now.

Unlike Deutsch’s CTCs, our P-CTCs always send pure states to pure states: they do not create entropy. As a result, P-CTCs provide a distinct resolution to Deutsch’s unproved theorem paradox, in which the time traveller reveals the proof of a theorem to a mathematician, who includes it in the same book from which the traveller has learned it (rather, *will* learn it). How did the proof come into existence? Deutsch adds an additional maximum entropy postulate to eliminate this paradox. By contrast, post-selected CTCs automatically solve it as shown in Fig. 2b through entanglement: because the circuit has no bias to one proof or another, the CTC creates an unbiased mixture of all possible ‘proofs.’

A user that has access to a closed timelike curve might be able to perform computations very efficiently: for pure state inputs, Deutsch’s CTCs permit the efficient solution of all problems in PSPACE [17, 18] (i.e. all problems that can be solved with polynomial space resources). However, Bennett *et al.* argued that this may be useless for computation, because CTCs decorrelate the outputs of the computation from its inputs stored elsewhere [19]. In contrast, Aaronson’s results on the power of post-selection in quantum computing imply that the P-CTCs considered here permit the efficient solution of problems in PP [20] (i.e. problems that a probabilistic polynomial Turing machine accepts with probability $\frac{1}{2}$ if and only if the answer is “yes.”) Although PP is a putatively less powerful class than PSPACE, P-CTCs are computationally very powerful: they do not decorrelate the inputs from the outputs and can efficiently solve NP-complete problems. Indeed, it is easy to see that P-CTCs can perform any computation a circuit of depth one.

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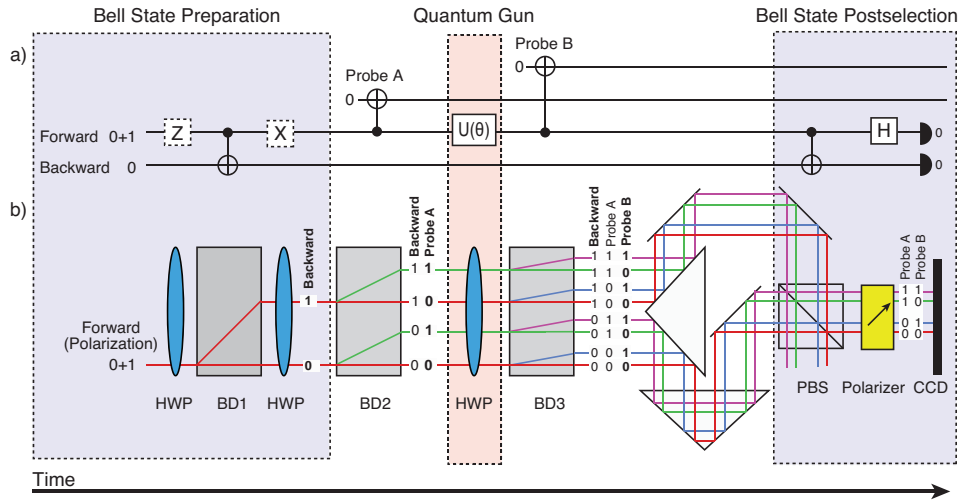


FIG. 3: Experiment to illustrate the P-CTC predictions of the grandfather paradox. a) Diagram of the quantum circuit. Using a CNOT gate sandwiched between optional Z and X gates, it is possible to prepare all of the maximally entangled Bell states. The Bell state measurement is implemented using a CNOT and a Hadamard. Each of the probe qubits is coupled to the forward qubit via a CNOT gate. b) Diagram of experimental apparatus. The polarization and path degrees of freedom of single photons from a quantum dot are entangled via a calcite polarization-dependent beam displacer (BD1), implementing the CNOT. Half-wave plates (HWP) before and after BD1 implement the optional Z and X gates. The state $|\phi^+\rangle$ is created by setting the angle of both HWPs to zero. To complete the teleportation circuit, the post-selection onto $|\phi^+\rangle$ is carried out by first recombining the path degrees of freedom on a polarizing beamsplitter (performing a CNOT gate between path and polarization) and then passing the photons through a calcite polarizer set to 45 degrees and detecting them on a cooled CCD. A rotatable HWP acts as a quantum gun, implementing the unitary $U(\theta) = e^{-i\theta\sigma_x}$. Removable calcite beam displacers (BD2 and BD3) couple the polarization qubit to two probe qubits encoded in additional spatial degrees of freedom. When the beam displacers are inserted in the setup, four spots on the CCD correspond to the probe states 11, 10, 01, and 00.

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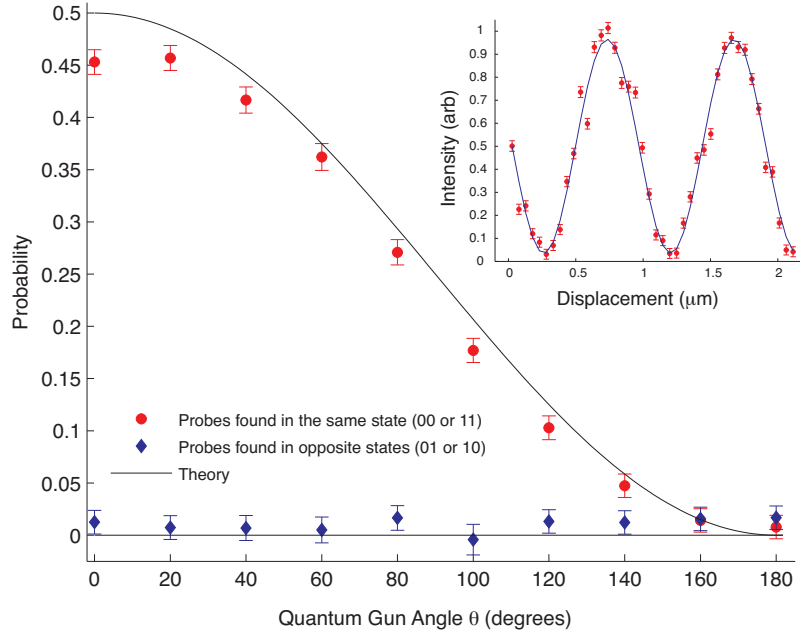


FIG. 4: Probability that time travel succeeds and the probes are found in the same state (red circles) or in opposite states (blue diamonds). When the quantum gun “misfires”, the polarization qubit is not flipped and the probe qubits are found in either the 00 or 11 state. As the accuracy θ of the quantum gun increases from 0 to π , the probability that the teleportation succeeds decreases. When the quantum gun “kills” the photon (flips the polarization qubit), the probes record opposite values (01 or 10). The probability that the probe qubits are found in either the 10 or 01 state is 0.01 ± 0.04 , indicating that the photons never succeed in travelling back in time and killing their former selves. Solid curves correspond to theoretical predictions. The discrepancy between theory and experiment when the probes are found in the same state is due to a $1.1 \pm 0.1^\circ$ mismatch between polarizers used in the state creation and measurement portions of the teleportation circuit. Data were collected for 6 seconds at each point. The error bars are due to photon counting statistics and background fluctuations from the cooled CCD. Inset: the teleportation loop constitutes a polarization interferometer. Its visibility was measured as $93 \pm 3\%$ by varying the phase (path-length difference) between the two paths, converting $|\phi^+\rangle$ to $(|00\rangle + e^{i\phi}|11\rangle)/\sqrt{2}$ before post-selecting on $|\phi^+\rangle$.