

PAPER • OPEN ACCESS

Parisi-Sourlas-like dimensional reduction of quantum gravity in the presence of observers

To cite this article: Dmitriy Podolskiy *et al* JCAP05(2021)048

View the [article online](#) for updates and enhancements.

You may also like

- [Comparison of non-Markovianity criteria in a qubit system under random external fields](#)

Maria Mannone, Rosario Lo Franco and Giuseppe Compagno

- [Tunable magnetic anisotropy in Cr-trihalide Janus monolayers](#)

Rehab Albaridy, Aurelien Manchon and Udo Schwingenschlögl

- [Primordial black holes in a dimensionally oxidizing Universe](#)

Konstantinos F. Dialektopoulos, Piero Nicolini and Athanasios G. Tzikas

Parisi-Sourlas-like dimensional reduction of quantum gravity in the presence of observers

Dmitriy Podolskiy,^a Andrei O. Barvinsky^b and Robert Lanza^c

^aHarvard University,
77 Avenue Louis Pasteur, Boston, MA 02115, U.S.A.

^bLebedev Physics Institute, Theory Department
Leninsky Prospekt 53, Moscow 117924, Russia

^cWake Forest University,
1834 Wake Forest Rd., Winston-Salem, NC 27106, U.S.A.

E-mail: Dmitriy_Podolskiy@hms.harvard.edu, barvin@td.lpi.ru,
robertlanza@outlook.com

Received November 20, 2020

Revised February 4, 2021

Accepted April 28, 2021

Published May 18, 2021

Abstract. We show that in the presence of disorder induced by random networks of observers measuring covariant quantities (such as scalar curvature) (3+1)-dimensional quantum gravity exhibits an effective dimensional reduction at large spatio-temporal scales, which is analogous to the Parisi-Sourlas phenomenon observed for quantum field theories in random external fields. After averaging over disorder associated with observer networks, statistical properties of the latter determine both the value of gravitational constant and the effective cosmological constant in the model. Focusing on the dynamics of infrared degrees of freedom we find that the upper critical dimension of the effective theory is lifted from $D_{\text{cr}} = 1 + 1$ to $D_{\text{cr}} = 3 + 1$ dimensions.

Keywords: modified gravity, initial conditions and eternal universe, alternatives to inflation, inflation

ArXiv ePrint: [2004.09708](https://arxiv.org/abs/2004.09708)



Contents

1	Introduction	1
2	Parisi-Sourlas-like dimensional reduction in Regge-Wheeler simplicial quantum gravity	3
3	Physical origin of possible Parisi-Sourlas-like dimensional reduction in Lorenzian quantum gravity	6
4	Fokker-Planck equation and its extensions in the two-noise model	11
5	Conclusion	15
A	Notes on von Neumann observers	17
B	Parisi-Sourlas dimensional reduction in non-renormalizable field theories in the presence of observer networks	17
C	Numerical simulations of field theories	19
C.1	Lattice simulations of Ising model and random field Ising model (RFIM)	19
C.2	Lattice simulations of pure and random field Z_2 gauge field theories	19
D	Deriving Starobinsky-Fokker-Planck equation using path integral approach	20
E	Non-locality in the one-noise model	21
E.1	Useful preliminary expressions and used notations	21
E.1.1	Slow roll parameters	21
E.1.2	Number of inflationary efoldings	21
E.2	Separating scalar field into IR and UV parts. Langevin equation	22
E.3	Commutation relations of the noise operators f_1 and f_2	23
E.4	Correlation functions of the noise operators f_1 and f_2	23
E.4.1	Relation to the Starobinsky's stochastic formalism	24

1 Introduction

Difficulty of establishing connection between general relativity and quantum mechanics has puzzled several generations of theoretical physicists starting with Albert Einstein [1]. At the heart of the problem is perturbative non-renormalizability of naively quantized general relativity [2, 3]. The theory becomes extremely sensitive to the choice of renormalization scheme used. This essentially means that the perturbative control over the behavior of the theory is lost.

This problem resurfaces on multiple levels and within any physical problem involving counting or accounting for quantum gravitational degrees of freedom. For example, in perturbative calculation of gravitational entropy associated with black hole horizon the numerical factor in front of the horizon area acquires infinite perturbative corrections (see [4] for the review). Thus, this prefactor becomes strongly dependent on the choice of regularization

scheme entangling the information loss paradox in quantum gravity [5] with the problem of its non-renormalizability. In quantum cosmology, where vacuum energy density essentially determines the expansion rate of the spacetime, perturbative corrections to its value are strongly scale dependent [6]. This implies in turn that even the sign of vacuum energy density is hard to determine with certainty. Again, behavior of the theory in the quantum cosmological setup is not controllable in both ultraviolet and infrared limit.

Starting with the Weinberg’s idea of asymptotic safety [7], it has been argued previously that canonical quantum gravity may be non-perturbatively renormalizable, with a UV fixed point. Numerical simulations of Regge-Wheeler simplicial quantum gravity (see [8] and references below), simulations employing dynamical triangulations [9–11] as well as functional renormalization group analysis [12] indeed all point towards the validity of this conclusion.¹ However, given that gravity is the weakest force [13], relying on the existence of a fixed point in a deeply UV regime feels unsatisfactory to us. Indeed, changing matter content of the theory would change in that case the location of the fixed point on its phase diagram possibly even removing it altogether for a specifically chosen matter content. Addressing the problem of non-renormalizability entirely within a perturbative domain, even given the extreme complexity of the problem, thus seems to us a more attractive possibility.

It has also become a common lore that a UV finite theory of gravity such as string theory would automatically guarantee avoidance of the problem of non-renormalizability. For example, counting microstates associated with extremal black hole horizon in string theory produces a correct finite answer $1/4$ for the numerical prefactor in black hole entropy [14].² On the other hand, it remains poorly understood (see for example [15]) if superstring theories provide *the unique* ultraviolet completion of naively quantized general relativity (GR). Might there be other UV completions which lead to the same controllable behavior in the continuum/infrared limit, completions which we are currently not aware of? In the former case, there should naturally exist a line of arguments which leads to emergence of effective string theoretic representation of the ultraviolet physics from an infrared effective GR setup. It would be desirable to demonstrate explicitly how a stringy behavior naturally emerges from this setup in the UV limit. We believe that the present work identifies a possible new research line along which such arguments can be obtained.

Namely, here we would like to argue that including “observers” which continuously measure such covariant quantities as scalar curvature (i.e., essentially probing the strength of gravitational interaction) and then averaging over disorder associated with a random network of these observers leads to an effectively de Sitter like behavior of the underlying theory of quantum gravity. Consequently, deep infrared behavior of the resulting $3 + 1$ -dimensional theory is effectively reduced to the one of a $1 + 1$ -dimensional theory. We identify a possible mapping between degrees of freedom in the original, $3 + 1$ -dimensional theory of quantum gravity and the ones in the effective $1 + 1$ -dimensional quantum theory obtained by averaging

¹Interestingly, simulations of both 4-dimensional simplicial quantum gravity and dynamically triangulated 4-dimensional quantum spacetime behave differently above and below this UV fixed point. Namely, an IR phase corresponds to a 4-dimensional AdS-like physics. On the other hand, a UV phase is characterized by a quasi-2-dimensional, branching polymer-like behavior in the UV phase. While this observation is often used in the community as a reason to discard results of these numerical simulations — IR behavior of spacetime we live in is manifestly dS-like rather than asymptotically AdS one — we shall argue below that this difference in behavior above and below the fixed point is actually physical.

²String theories are UV complete by construction and contain a scalar-tensor general relativity in their low energy effective actions. In this sense, they represent self-consistent UV completions of the naively quantized general relativity and guarantee UV completeness for the estimations of quantum gravitational effects.

over disorder associated with the presence of “observers” and taking the long wavelength limit. The identified mapping is reminiscent of the celebrated Parisi-Sourlas dimensional reduction known to take place in field theories with global and gauge symmetries in the presence of random external fields [16].

Finally, we also argue that the effective action of the emergent $1+1$ -dimensional theory coincides with the Liouville scalar theory, i.e., essentially, the theory of two-dimensional quantum gravity [17, 18]. This result has a potential to provide the missing link between naively quantized general relativity and string theory. Importantly, it also explains why observed dimensionality of spacetime which we live in is $D = 3 + 1$.

We deem these observations generally interesting also because the described setup, quantum gravity with disorder, represents a rare case in theoretical physics when the presence of observers drastically changes behavior of observable quantities themselves not only at microscopic scales but also in the infrared limit, at very large spatio-temporal scales. Namely, in the absence of observers the background of the $3+1$ -dimensional quantum gravity remains unspecified. Once observers are introduced, coupled to the observable gravitational degrees of freedom and integrated out, the effective background of theory becomes de-Sitter like. Rather than being a fundamental constant of the theory, the characteristic curvature of effective cosmological constant is determined by the intrinsic properties of “observers” such as the strength of their coupling to gravity and distribution of observation events across the fluctuating spacetime. Physical observers thus play a critically important role for our conclusions implying a necessity of proper description of observer, observation event and interaction between observers and the observed physical system for theoretical controllability of the very physical setups being probed.

The text of the manuscript is organized as follows. Section 2 is devoted to a numerical study of simplicial Regge-Wheeler (Euclidean) quantum gravity in the presence of random Gaussian field coupled to scalar curvature. We argue that the theory exhibits an analogue of Parisi-Sourlas dimensional reduction after averaging over quenched disorder associated with events of gravitational field probing. In section 3 we represent theoretical arguments explaining results of this study and pointing towards their validity in a continuous Lorentzian quantum theory. The section 5 is devoted to the outline of obtained results and a brief discussion of several analogies of phenomena observed here and the ones realized in condensed matter physics. Finally, appendices include details of numerical simulations of several quantum field theories which we used as a pilot study for the subsequent work on quantum gravity. They also contain a more detailed theoretical derivation of the results of section 3.

2 Parisi-Sourlas-like dimensional reduction in Regge-Wheeler simplicial quantum gravity

Following the approach by Regge and Wheeler [19–21, 27, 28], we consider a pure 4-dimensional Euclidean quantum gravity with a cosmological constant.³

³Regge-Wheeler simplicial gravity [21, 28] might very well be a very distant cousin of the naively quantized general relativity. For example, it is not yet entirely clear if (a) the theory preserves local gauge invariance in the number of dimensions $D > 2$ [29], (b) Euclidean setup critical for the theory is sufficient to capture essentially Lorentzian behavior of true Einstein gravity including, in particular, its gravitational instability, and (c) the theory actually contains a massless spin-2 particle in the spectrum of its low energy perturbations [30]. However, at the moment it remains the best setup which we can use attempting numerical studies of quantum general relativity.

We are interested to determine possible changes in behavior of observables of the theory in the presence of an extra ingredient: von Neumann observers randomly distributed across the fluctuating spacetime and measuring the strength of gravitational self-interaction [22, 23] (please see appendix for the discussion). Observational events associated with their activity can be modeled by the term

$$\sqrt{g}\phi(x)R(x) = 2 \sum_{h \supset x} \phi_x \delta_h A_h \quad (2.1)$$

in the Lagrangian density of the discrete simplicial gravity. In the expression (2.1) the left-hand side of the equality represents a continuum version of the theory with the scalar curvature $\sqrt{g}R(x)$ calculated at the point of spacetime x . The right-hand side represents a corresponding discretized version with the sum running over hinges of simplices crossing the point x and serving as building blocks of spacetime. A_h is the area of the hinge, δ_h is the associated deficit angle defined according to $\delta_h = 2\pi - \sum_{\text{blocks meeting at } \theta} \theta$ and θ is the corresponding dihedral angle. The field ϕ representing von Neumann observers is a source of quenched disorder in the theory which we consider Gaussian distributed in our simulations.

As was briefly mentioned in the Introduction, since the Regge-Wheeler theory possesses a UV fixed point in the number of dimensions $D = 2, 3, 4$ [21], the problem of comparing observables in the presence of disorder (2.1) and without it is reduced to the problem of comparing critical exponents of the theory at the fixed point $k = k_c$. In particular, we were interested in the dependence of the universal critical exponent ν on the background space dimensionality. As usual, we define the critical exponent ν through the average space curvature

$$\frac{\langle \int d^D x \sqrt{g} R \rangle}{\langle \int d^D x \sqrt{g} \rangle} \sim (k_c - k)^{D\nu-1}, \quad (2.2)$$

where $k = 1/8\pi G$ and k_c represents the critical point of the theory. Here, the integration is performed over the D -dimensional spacetime in a particular simplicial realization of the lattice theory. The average is taken over the statistical ensemble of simplices, i.e., all realizations in this ensemble. In addition, when observers are introduced, averaging also includes the one over disorder associated with observers (that is, averaging over the ensemble of statistical realizations of observers). In this sense, the denominator of the eq. (2.2) represents the D -volume of the spacetime averaged over realizations of both ensembles, while the numerator — the average scalar curvature. Both integrals are well defined on a finite lattice.

The exponent ν is directly related to the derivative of the beta function for the gravitational constant near the ultraviolet fixed point according to $\beta'(k_c) = -\nu^{-1}$. Namely, in $D = 2 + \epsilon$ space dimensions one has (assuming free gravity with a cosmological constant) [24–26]

$$\frac{1}{8\pi k_c} = \frac{3}{50}\epsilon - \frac{9}{250}\epsilon^2 + \dots, \quad (2.3)$$

$$\nu^{-1} = -\beta'(k_c) = \epsilon + \frac{3}{5}\epsilon^2 + \dots \quad (2.4)$$

To approach the problem in question, we have performed Monte-Carlo simulations of simplicial Euclidean quantum gravity in $D = 4$ space dimensions on hypercubic lattices of sizes $L = 4$ (256 sites, 3840 edges, 6144 simplices), 8 (4096 sites, 6144 edges, 98304 simplices) and 16 (65536 sites, 983040 edges, 1572864 simplices). In all simulations, the topology was fixed to be the one of 4-torus. No fluctuations of topology were allowed. The bare cosmological constant was also fixed to 1 (since the gravitational coupling is setting the overall

length scale in the physical problem). To establish efficient thermalization of the system in our numerical experiment (in the absence of disorder) we have investigated behavior of the system at 20 different values of k . For $L = 16$ hyper-lattice 33000 consequent configurations were generated for every single realization of disorder, for $L = 8$ hyper-lattice — 100000 configurations and for $L = 4$ hyper-lattice — 500000 consequent configurations. Obtained dependence of the average curvature (2.2) was then fit to the singular dependence on k to determine the values of critical gravitational coupling k_c and the critical exponent ν . In the absence of disorder (setting all couplings to the disorder field ϕ_k to 0) we found for the $L = 4$ hyper-lattice $k_c = 0.067(3)$, $\nu = 0.34(5)$, for the $L = 8$ hyper-lattice — $k_c = 0.062(5)$, $\nu = 0.33(6)$ and for the $L = 16$ hyper-lattice — $k_c = 0.061(7)$, $\nu = 0.32(9)$; a relatively weak dependence of the fixed point scale k_c on L pointed out towards efficient thermalization of the employed Euclidean lattice system.

We repeated the same procedure for 10000 different realizations of the random disorder ϕ_k . Fitting dependence of the average curvature on k for configurations averaged over disorder, we found the value of k_c (post disorder averaging) to be $k_c \approx 0.03 \pm 0.12$, in principle consistent with $k_c = 0$ (compare with $k_c \approx 0.07$ in the case without disorder). We have found that the value $\nu^{-1} = 0.01 \pm 0.06$ for the $L = 4$ hyper-lattice, $\nu^{-1} = 0.02 \pm 0.05$ for the $L = 8$ hyper-lattice and $\nu^{-1} = 0.02 \pm 0.04$ for the $L = 16$ hyper-lattice (compare with $\nu^{-1} \approx 3$, which holds approximately in the case without disorder).

In principle, both of these observations (vanishing of k_c and ν^{-1}) — but especially the second one — are consistent with a Parisi-Sourlas-like dimensional reduction in the presence of disorder (2.1). Indeed, it has been argued previously (see for example [30]) that $\nu \approx \frac{1}{D-1}$ for large D , while $\nu = \infty$ exactly for $D = 2$. If an analogue of Parisi-Sourlas dimensional reduction holds also for quantum gravity, this naturally implies that the upper critical dimension of gravity ($D = 2$ in the absence of disorder) is lifted to 4 in the presence of a random network of von Neumann detectors performing measurements of scalar curvature.⁴ An effective low dimensionality emerging in simulations of simplicial quantum gravity has been previously also reported in [28, 31] where it has been argued that the UV phase of the theory features an effective dimensional reduction with polymer-like behavior of the correlation functions of observables, while its IR physics is smooth with effectively Euclidean AdS (EAdS) background. Vanishing of the critical value k_c after averaging over quenched disorder (2.1) would in turn force one to think that the UV phase becomes the only accessible one across all scales k , naively implying unphysical behavior of the theory in the presence of quenched disorder. We shall argue in the next section that the observed behavior is fully physical and, in sense, a natural one which should be expected from the quantum theory of gravity in the presence of quenched disorder.

Finally, we note in passing that k_c vanishing after averaging over disorder in gravity also seems analogous to a phenomenon which has already been observed in field theories with quenched disorder: for example, the 2nd order phase transition of Ising model (reduced to $\lambda\phi^4$ scalar field theory in the continuum limit) is reached at finite temperature T_c in the absence of disorder and at $T = T_c$ in the presence of random external field [63].

⁴One can ask what happens in simplicial Euclidean quantum gravity (with a quenched disorder) at $D_{\text{cr}} > 4$ and at $D = 3$? If the analogy with behavior of field theories in external fields holds for gravity completely, we expect the theory of simplicial ($D = 5$)-dimensional quantum gravity with a quenched disorder to be equivalent to a 3-dimensional theory without such disorder etc. On the other hand, Parisi-Sourlas correspondence would break down at $D = 3$ in a similar fashion as it happens in $D = 3$ random field Ising model, see discussion in the appendix. We leave this question to the future study.

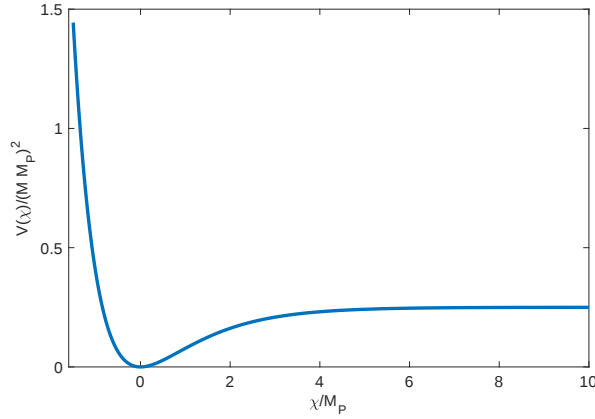


Figure 1. Effective potential $V(\chi) = \frac{M^2 M_P^2}{4} (1 - \exp(-\sqrt{2/3}\chi/M_P))^2$ of the scalar mode $\chi = \sqrt{3/2}M_P \log(1 + \frac{R}{M^2})$, where $M = \Delta^{-1/2}M_P$, related to spacetime curvature in the Jordan frame. After averaging over quenched disorder, all not-trivial correlations of gravitational degrees of freedom are represented by correlation functions of χ .

3 Physical origin of possible Parisi-Sourlas-like dimensional reduction in Lorenzian quantum gravity

The observed effect of dimensional reduction in Regge-Wheeler simplicial quantum gravity can be understood (and possibly explained) using the following theoretical arguments.

The continuum limit of the theory (2.1) (assuming that it exists) is expected to correspond to a scalar-tensor Euclidean gravity, where the “dilaton” field ϕ is sufficiently massive, so that its arbitrary configuration in the world volume of the theory can be considered a quenched disorder. The physical idea here is that observers/detectors have a “lifespan” much shorter than the deep infrared scales which we are interested in. Thus, they can be considered as point-like observation “events”. The distribution of these “events” is given by the statistical properties of the field ϕ fluctuating across the spacetime. Averaging over the associated quenched disorder (i.e., over the distribution of ϕ), we essentially assume that observers exchange information between each other, and these information exchange processes again happen at time scales much shorter than the infrared scales of interest. Given that, for any given observer the measurement of the gravitational field strength is Bayes-conditioned on the results of similar measurements by other observers in the network. In this sense, the states realized in the effective theory obtained by averaging over distribution of ϕ are “consensus” states.

If the disorder associated with observers is Gaussian, the partition function of the continuum version of the theory is then given by

$$Z = \int \mathcal{D}\phi \int \frac{\mathcal{D}g}{\mathcal{D}f} \exp \left(- \int d^4x \sqrt{g} \left((M_P^2 + \phi)R + \frac{\phi^2}{2\Delta} \right) \right), \quad (3.1)$$

where the integration measure in the path integral over space metric g is assumed to be invariant with respect to arbitrary diffeomorphisms (hence the division by the volume of Diff group $\mathcal{D}f$) and the parameter Δ determines the width of the distribution of the field ϕ describing observers. Note that the field ϕ has the dimension of mass squared, so that the parameter Δ is dimensionless.

Integrating over all possible realizations of ϕ and performing analytic continuation to spacetimes with Lorentz signature,,⁵ one obtains an effective $f(R)$ -theory of gravity with partition function $Z \sim \int \frac{\mathcal{D}g}{\mathcal{D}f} \exp(-i \int d^4x \sqrt{-g} f(R))$ and

$$f(R) \sim M_P^2 R + \frac{\Delta R^2}{2}. \quad (3.2)$$

(As an interesting digression, we note that one does not really need to introduce the “bare” gravitational interaction coupling M_P at all if ϕ has a non-zero average over configurations of ϕ ; in the latter case, the effective Planck constant simply reads $M_{P,\text{eff}}^2 = \langle \phi \rangle$.)

This theory manifestly admits de Sitter-like solutions for all possible values of its parameters [32]. Such solutions represent dynamical attractors in the phase space of the theory. Indeed, switching from the Jordan frame to the Einstein frame in the $f(R)$ -theory of gravity, one finds that the theory (3.2) is effectively equivalent to a theory of gravity coupled to a scalar field

$$\chi \sim \sqrt{3/2} M_P \log \left(1 + \frac{\Delta R}{M_P^2} \right), \quad (3.3)$$

where R is the scalar curvature of spacetime in the original $f(R)$ -theory. As always in analysis of an inflationary theory, we are interested in the case of super-Planckian χ , meaning that $\frac{M_P^2}{\Delta} \ll R \ll M_P^2$ (i.e., the dimensionless parameter Δ is very large, so that the distribution of couplings of gravitational detectors to R has a single narrow peak).

The potential of this effective scalar field in the Einstein frame is given by [33–37]

$$V(\chi) \sim \frac{3M_P^4}{4\Delta} (1 - \exp(-\sqrt{2/3} \chi/M_P))^2, \quad (3.4)$$

which reduces to the potential of chaotic inflation at small $\chi \ll M_P$ and a potential quickly approaching a constant asymptotics at $\chi \gg M_P$. In the regime of interest $\Delta \gg 1$ the energy scale is well below M_P .

We also note that the case $\chi < 0$ corresponds to an anti de Sitter-like regime with $\langle R \rangle < 0$ in the Jordan frame, while the regime $\chi > 0$ — to a de Sitter-like physics. For $\chi < 0$, $|\chi| \gg M_P$ corresponds to a collapsing solution in the Einstein frame but anti-de Sitter physics in the Jordan frame with R bounded from below by M_P^2/Δ . Again, physics of the system interestingly depends on the statistical properties of the distribution of observers and observation events in the spacetime. Both the effective value of the Planck constant ($M_P^2 \sim \langle \phi \rangle$) and the one of the effective cosmological constant are determined by statistical properties of the observer networks in this model.

Focusing attention to the case in question with $\chi > 0$, we would like to integrate out sub-horizon fluctuations of the effective field χ (such fluctuations can be considered Gaussian in the first approximation due to applicability of EFT approximation for gravitational degrees of freedom in the UV). Once sub-horizon physics is integrated out, one arrives to the physical picture of an inflationary self-reproducing universe. The only survived “coordinates” here are the number of inflationary efoldings (log of scale factor) and an effective scalar field χ (essentially, a log of scalar curvature in the Einstein frame). In this sense, the originally $(3+1)$ -dimensional theory becomes effectively 2-dimensional in the infrared. Let us show in details that this is indeed the case using stochastic inflationary formalism [38–49] and

⁵The question how such analytic continuation should be performed technically is far from trivial. Here for the sake of simplicity we shall follow the naive prescription for the Wick rotation $t \rightarrow -it$.

ignoring gravitational vector and tensor modes which do not contribute to quasi-de Sitter gravitational entropy [50] and thus do not influence strongly infrared dynamics of the theory.

Namely, separating the field χ into the subhorizon and superhorizon parts, one can write:

$$\chi(t, x) = \chi_{IR}(t, x) + \frac{1}{(2\pi)^{3/2}} \int d^3k \cdot \theta(k - \epsilon a H) (a_k \phi_k(t) \exp(-ikx) + \text{h.c.}) + \delta\chi. \quad (3.5)$$

Here $a(t)$ is the scale factor of de Sitter spacetime, H is the corresponding Hubble constant, $\theta(\dots)$ is the Heaviside step-function, the modes $\phi_k(t) = \frac{H}{\sqrt{2k}}(\tau - \frac{i}{k}) \exp(ik\tau)$ correspond to the Bunch-Davies de Sitter invariant vacuum of a free massless scalar field, $\tau = \int \frac{dt}{a(t)}$ (in what follows, we will be working in the uniform N -gauge), ϵ is a small number such that $\epsilon \ll 1$ (which determines a notation for separating superhorizon modes from the subhorizon ones) and $\delta\chi$ can be neglected in the leading order with respect to H/M_P i.e., when slow roll parameters remain small. This correction encodes the difference between the uniform N gauge and the spatially flat gauge used to derive the correlation properties of the noise $f(\tau, x)$ below [51]. Substituting this decomposition back into the equation of motion for the field χ on de quasi-Sitter background, one obtains the equation for the infrared part of the field χ_{IR} :

$$\frac{d\chi_{IR}}{d\tau} = -\frac{1}{3H^2} \frac{dV}{d\chi_{IR}} + \frac{f(\tau, x)}{H}, \quad (3.6)$$

where a composite operator

$$f(\tau, x) = \frac{\epsilon a H^2}{(2\pi)^{3/2}} \int d^3k \cdot \delta(k - \epsilon a H) \cdot \frac{(-i)H}{\sqrt{2}k^{3/2}} [a_k \exp(-ikx) - a_k^\dagger \exp(ikx)]$$

has the correlation properties

$$\langle f(\tau, x) f(\tau', x) \rangle = \frac{H^4}{4\pi^2} \delta(\tau - \tau'),$$

if the average is taken over the Bunch-Davies vacuum state. Another very important property of this operator is that its self-commutator vanishes, and thus the equation (3.6) can be considered a stochastic differential equation for the quasi-classical but stochastically distributed long-wavelength field χ_{IR} (from now on, we shall drop the index IR always implying that the infrared, superhorizon part of the field χ is considered).

One then obtains an effective Fokker-Planck equation (see for example appendix D) corresponding to the Langevin equation (3.6) for the probability $P(\tau, \chi)$ to measure a given value of the background/infrared scalar field χ in a given Hubble patch:

$$\frac{\partial P}{\partial \tau} \approx \frac{1}{3\pi M_P^2} \frac{\partial^2}{\partial \chi^2} (VP) + \frac{M_P^2}{8\pi} \frac{\partial}{\partial \chi} \left(\frac{1}{V} \frac{dV}{d\chi} P \right), \quad (3.7)$$

where $\approx$ implies that the equation (3.7) by itself is an approximation (we made a number of simplifications during its derivation such as neglecting the subdominant contribution $\delta\chi$ in the expansion (3.5), assuming slow roll of the field χ and neglecting self-interaction of the field χ at subhorizon scales). We expect that the eq. (3.7) holds on average and only approximately, and model it by including an additional term $F(\tau, \chi)$ to its right-hand side, again quasi-classical but stochastic (see the next section):

$$\frac{\partial P}{\partial \tau} = \frac{1}{3\pi M_P^2} \frac{\partial^2}{\partial \chi^2} (VP) + \frac{M_P^2}{8\pi} \frac{\partial}{\partial \chi} \left(\frac{1}{V} \frac{dV}{d\chi} P \right) + F,$$

Taking into account the smallness of this term, assuming its Gaussianity

$$\langle F(\tau, \chi) F(\tau', \chi') \rangle = A \delta(\tau - \tau') \delta(\chi - \chi')$$

and integrating it out, we finally conclude that the infrared dynamics of the theory (3.1) is being essentially determined by the partition function

$$Z_{IR} = \int \mathcal{D}P \exp(-\mathcal{W}),$$

where the effective action $\mathcal{W}$ of the theory is given by

$$\mathcal{W} = \int d\tau d\chi \frac{1}{A} \left(-\frac{\partial P}{\partial \tau} + \frac{1}{3\pi M_P^2} \frac{\partial^2}{\partial \chi^2} (VP) + \frac{M_P^2}{8\pi} \frac{\partial}{\partial \chi} \left(\frac{dV}{V d\chi} P \right) \right)^2. \quad (3.8)$$

It is now instructive to use the de Sitter “entropy” S defined according to the prescription $P(\tau, \chi) = \exp(S(\tau, \chi))$ instead of the probability distribution P . One motivation for this substitution is the fact that the distribution function $P(\tau, x)$ converges to

$$P(\tau \rightarrow \infty, \chi) \sim \frac{1}{V(\chi)} \exp \left(\frac{3M_P^4}{8V(\chi)} \right)$$

in the limit $\tau \rightarrow \infty$, where the expression in the exponent coincides exactly with the gravitational entropy of de Sitter space.⁶ As we shall see below, there are other advantages of using S instead of P .

One finds after the substitution

$$\begin{aligned} \mathcal{W} = \int d\tau d\chi \left(\frac{e^{2S}}{A} \left[-\frac{\partial S}{\partial \tau} + \frac{V}{3\pi M_P^2} (S'' + (S')^2) + \left(\frac{2V'}{3\pi M_P^2} + \frac{M_P^2}{8\pi} (\log V)' \right) S' + \dots \right. \right. \\ \left. \left. \left(\frac{V''}{2\pi M_P^2} + \frac{M_P^2 (\log V)''}{8\pi} \right) \right]^2 - S \right), \end{aligned} \quad (3.9)$$

where prime denotes partial differentiation with respect to the field χ . The appearance of the last term is due to the Jacobian in the measure of functional integration emerging after the change of functional variables.

Substituting the particular form of the potential (3.4) of interest for us to the expression (3.9), we obtain

$$\mathcal{W} = \int d\tau d\chi \left[\frac{e^{2S}}{A} \left(-\frac{\partial S}{\partial \tau} + \frac{M_P}{4\pi} \sqrt{\frac{2}{3}} z \frac{\partial S}{\partial \chi} - \frac{z}{6\pi} \right)^2 + \dots \right], \quad (3.10)$$

where $z = \exp \left(-\sqrt{\frac{2}{3}} \frac{\chi}{M_P} \right)$. In the quasi-de Sitter limit $S = S_0 + \delta S$, $\delta S \ll S_0$, the potential term in this action coincides with the one of a Liouville-like theory of the “field” S in a two-dimensional spacetime spanned by the coordinates (τ, χ) , i.e., the two-dimensional theory of quantum gravity [17, 18] with the “field” S playing the role of the conformal mode of the 2-dimensional spacetime metric.

⁶Generally speaking, this holds only for potentials $V(\chi)$ positive for any value of χ . For the potential $V(\chi)$ depicted on the figure 1 this is not the case: as $\chi \rightarrow 0$, slow roll approximation breaks down, and inflation ends, which makes the distribution function $P(\tau, \chi)$ non-normalizable, with $P \rightarrow 0$ as $\chi \rightarrow 0$.

In other words, in the absence of anisotropic stress covariant observables in quantum gravity can be expressed in terms of correlation functions of the scalar curvature (in one-to-one correspondence with the scalar degree of freedom χ in the Einstein frame) according to the prescription

$$\langle \chi^n \rangle \sim \langle (M_P \log \left(\frac{R}{M^2} \right))^n \rangle \sim \int d\chi \cdot \chi^n \int \mathcal{D}S \exp(-\mathcal{W}), \quad (3.11)$$

where the effective action in the path integration (3.11) is given by the expression (3.10). Therefore, the limit $\log a \rightarrow \infty$ of the integrand in eq. (3.11) can be thought of as a ground state of the theory of gravity (3.2).

To formalize the map (3.11) a bit clearer, we can write averages of any observable $\langle \mathcal{O} \rangle(t)$ at the time $t \sim H^{-1} \log(a)$ as

$$\langle \mathcal{O}(\chi) \rangle(t) = \int d\chi \mathcal{O}(\chi) P(\chi, t), \quad (3.12)$$

where the partition function P satisfies the Fokker-Planck equation

$$E(\dot{P}, \partial_\chi P, \chi) \equiv -\dot{P} + \hat{H}P = 0. \quad (3.13)$$

We thus have a chain of transformations

$$\langle \mathcal{O}(\chi) \rangle(t) = \int d\chi \mathcal{O}(\chi) P(\chi, t) = \int d\chi \mathcal{O}(\chi) \int D\bar{P} \bar{P}(\chi, t) \delta[E(\dot{\bar{P}}, \partial_\chi \bar{P}, \chi)], \quad (3.14)$$

where the Jacobian for transformation between E and P is disregarded for simplicity. The functional delta-function in the last integral on the right is effectively regulated by the small parameter $A \rightarrow 0$ according to

$$\begin{aligned} \delta[E(\dot{\bar{P}}, \nabla \bar{P}, \chi)] &= \prod_{t, \chi} \delta(E(\dot{\bar{P}}(t, \chi), \partial_\chi \bar{P}(t, \chi), \chi)) \equiv \prod_x \delta(E(\nabla \bar{P}(x), \nabla \bar{P}(x), \chi)) \\ &= \exp \left(-\frac{1}{A} \int d^2x E^2(\nabla \bar{P}(x), \nabla \bar{P}(x), \chi) \right). \end{aligned} \quad (3.15)$$

Here we introduced 2D coordinates $(x_0, x_1) = (\log(a), \chi)$, and $\nabla = \partial_0, \partial_1$ collects all 2D derivatives. Thus we have (dropping bar over functional integration variable P)

$$\begin{aligned} \langle \mathcal{O}(\chi) \rangle(t) &= \int D\bar{P} \exp \left(-\frac{1}{A} \int d^2x E^2(\nabla \bar{P}(x), \nabla \bar{P}(x), \chi) \right) \int d\chi \mathcal{O}(\chi) \bar{P}(\chi, t) \\ &= \int DP \exp \left(-\frac{1}{A} \int d^2x E^2(\nabla P(x), \nabla P(x), \chi) \right) \int d^2y \mathcal{O}(y^1) P(y) \delta(y^0 - t). \end{aligned}$$

The probability P to measure a given value of χ in a given Hubble patch is then reparametrized as $P \sim \exp(S)$, and S become the field variable of interest for us. (Note that in the quasi-de Sitter limit S coincides with the gravitational entropy of de Sitter space in the limit $\log(a) \rightarrow 0$.) The mapping dictionary of duality between 2D side and 4D side for the action in quantum measure and for observables of interest is then defined according to the prescription

$$\exp \left(-\frac{1}{A} \int d^2x E^2(\nabla \bar{P}(x), \nabla \bar{P}(x), \chi) \right) \Leftrightarrow \exp \left(-\text{Polyakov action} \right) \quad (3.16)$$

$$\mathcal{O}(\chi) \Leftrightarrow \int d^2y \mathcal{O}(y^1) P(y) \delta(y^0 - t), \quad (3.17)$$

The physical reason why the dimensional reduction has effectively realized in the theory (3.2) and its analytic continuation to Lorentzian spacetimes is simple. Once the dynamics of relevant degrees of freedom is coarse-grained to comoving spatio-temporal scales $\sim H_0^{-1} \sim \Lambda^{-1/2}$,⁷ the global structure of spacetime is represented by a set of causally unconnected Hubble patches. Expectation values and correlation functions of the field χ are determined by a stochastic process generated by the Langevin equation (3.6). The values of χ in different Hubble patches are completely independent of each other, and thus the spatial dependence of χ becomes largely irrelevant.

We have argued that the 4-dimensional gravity with a quenched “dilaton” χ becomes reduced to an effectively two-dimensional theory in the deep infrared limit (of large spatio-temporal coarse-graining), where coordinate mapping of the fluctuating spacetime is given in terms of the number of efoldings $\tau = \log a$ and the effective scalar degree of freedom χ related to the large-scale curvature of spacetime in the Einstein frame according to the prescription (3.3). We emphasize that the physical scales at which this description becomes efficient coincide and exceed the scales of eternal inflation from the point of view of a subhorizon observer, thus effectively regularizing the structure of the theory in this deep IR limit. Tensor and vector degrees of freedom present in the metric for the subhorizon observer are effectively integrated away and do not contribute to the infrared structure of the correlation functions of observables in the theory. When the probe scale approaches the cosmological horizon scale, this effectively 2D physics has to be matched to an effective 4D field theory description of gravitational degrees of freedom. It is quite clear from the setup how it has to be done physically. Namely, effective subhorizon 4D degrees of freedom including vector and tensor ones are propagating on the stochastic background with large scale statistical properties effectively determined by the Liouville physics described above.

4 Fokker-Planck equation and its extensions in the two-noise model

In this section, we shall derive the effective action (3.8) used above, albeit in a schematic fashion, and estimate dependence of the parameter A in (3.8) on δ and slow roll parameters.

As was discussed previously, the inflationary Fokker-Planck equation holds its canonical celebrated form (3.7) only in the regime $\delta \rightarrow 0$, $\epsilon_H \rightarrow 0$, which does not necessarily hold anywhere except very close to the de Sitter spacetime geometry. Moreover, even for geometries globally close to dS_4 one might be interested in behavior of the IR effective theory under different values of parameter δ separating IR and UV physics. At this point, one would only be aware of the fact that the theory approaches the regime $A \rightarrow 0$ with Fokker-Planck-like dynamics of $P(\chi, N)$ at $\delta \rightarrow 0$, which is entirely independent of δ . In short, we would like to derive extension of this equation which would hold to first order in slow roll parameters ϵ_H , η_H and, ideally, to order in δ higher than first.

First of all, one notes that the one-noise stochastic model for the infrared dynamics of the scalar field in the inflationary spacetime (3.6)–(3.7) cannot be used for this derivation as it produces manifestly non-local results, see appendix E. This non-locality stems from the presence of additional degree of freedom which is integrated out to obtain the effective theory (3.6)–(3.7) in the case of generic ϵ_H , δ . It can be shown that this degree of freedom

⁷As we are interested in the continuum limit of the theory (2.1), it is natural to study exactly this case.

can be accounted for if we consider a two-noise model similar to the one introduced in [40]:

$$\frac{d\Phi}{dN} = \frac{v}{H} + \sigma, \quad (4.1)$$

$$\frac{dv}{dN} = -3v - H^{-1} \frac{\partial V}{\partial \Phi} + \tau, \quad (4.2)$$

where

$$\sigma(N, \mathbf{x}) = \frac{1}{H} \int \frac{d^3k}{(2\pi)^{3/2}} \delta(k - \delta a H) \left(a_k \phi_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.} \right) \quad (4.3)$$

$$\tau(N, \mathbf{x}) = \frac{1}{H} \int \frac{d^3k}{(2\pi)^{3/2}} \delta(k - \delta a H) \left(a_k \dot{\phi}_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.} \right) \quad (4.4)$$

The expressions for the modes ϕ_k , $\dot{\phi}_k$ have to be derived under assumption of finite (but small ϵ_H). Importantly, to the first order in small roll parameters we can keep ϵ_H constant (see appendix E). The equation for the modes $u_k = a\phi_k$ then has the form

$$0 = u_k'' + \left(k^2 - \frac{a''}{a} + m^2 a^2 \right) u_k = u_k'' + \left(k^2 - \left(2 - \epsilon_H - \frac{m^2}{H^2} \right) H^2 a^2 \right), \quad (4.5)$$

where m is the effective mass of the scalar field. To the leading order, we have $\epsilon_H \approx \frac{m^2}{3H^2}$, and thus $2 - \epsilon_H - \frac{m^2}{H^2} \approx 2 - 4\epsilon_H = 2(1 - \epsilon_H)$. On the other hand, again, $a \approx -\frac{1}{H\eta}(1 + \epsilon_H) + \mathcal{O}(\epsilon_H^2)$, and we finally obtain that (to the leading linear order in slow roll parameters ϵ_H) the field u_k satisfies the free massless field equation

$$u_k'' + \left(k^2 - \frac{2}{H^2 \eta^2} \right) = 0, \quad (4.6)$$

with its properly normalized solution given by

$$u_k(\eta) = -\frac{1}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right) \exp(-ik\eta) + \mathcal{O}(\epsilon_H^2). \quad (4.7)$$

Thus, to the first order in ϵ_H the mode ϕ_k is given by

$$\phi_k(\eta) = \frac{H(1 + \epsilon_H)}{\sqrt{2k}} \left(\eta - \frac{i}{k} \right) \exp(-ik\eta) + \mathcal{O}(\epsilon_H^2) \quad (4.8)$$

Differentiating this expression with respect to world time $t = \int \frac{dN}{H}$ we find

$$\frac{d\phi_k(\eta)}{dt} = \frac{ikH^2\eta^2}{\sqrt{2k}} e^{-ik\eta} - \frac{\epsilon_H H^2}{\sqrt{2k}} \left(\eta - \frac{i}{k} \right) e^{-ik\eta} + \mathcal{O}(\epsilon_H^2). \quad (4.9)$$

The correlation functions of the noise terms $\sigma(N)$ and $\tau(N)$ (as usual, we are interested only in the behavior of correlation functions in the same Hubble patch parametrized by the same “coarse-grained” spatial point $\mathbf{x}$) are in turn found to be

$$\langle \sigma(N) \sigma(N') \rangle \approx \frac{H^2}{4\pi^2} \delta(N - N') \left(1 + 3\epsilon_H + \delta^2 \right), \quad (4.10)$$

$$\langle \tau(N) \tau(N') \rangle \approx \frac{\delta^2 H^4}{4\pi^2} \delta(N - N') \left(\delta^2 + 2\epsilon_H \right) \quad (4.11)$$

$$\langle \tau(N) \sigma(N') \rangle \approx \delta(N - N') \frac{H^3}{4\pi^2} (\epsilon_H + \delta^2 - i\epsilon_H \delta) \quad (4.12)$$

(the latter two correlation functions vanish in the limit $\delta \rightarrow 0$), and the correlation function $\langle \sigma(N)\tau(N') \rangle$ is related to (4.12) by complex conjugation. Mixed $\tau\sigma$ correlators also end up suppressed by either powers of δ or ϵ_H . Note in this respect that one has to be careful taking one of the limits $\delta \rightarrow 0$ or $\epsilon_H \rightarrow 0$ first (compare for example to [40]). Two limiting cases are of special interest:

- (a) **Quasi-de Sitter limit.** $\epsilon_H \ll \delta^2 < 1$, the case considered in the appendix E with ϵ_H negligible but keeping all orders in δ

$$\begin{aligned}\langle \sigma(N)\sigma(N') \rangle &\approx \frac{H^2}{4\pi^2}(1 + \delta^2)\delta(N - N'), \\ \langle \tau(N)\tau(N') \rangle &\approx \frac{\delta^4 H^4}{4\pi^2}\delta(N - N') \\ \langle \tau(N)\sigma(N') \rangle &\approx \frac{\delta^2 H^3(1 + i\delta)}{4\pi^2}\delta(N - N')\end{aligned}$$

and

- (b) **“Deep IR physics” or Nambu-Sasaki limit.** $\delta^2 \ll \epsilon_H \ll 1$:

$$\begin{aligned}\langle \sigma(N)\sigma(N') \rangle &\approx \frac{H^2}{4\pi^2}(1 + 3\epsilon_H)\delta(N - N'), \\ \langle \tau(N)\tau(N') \rangle &\approx \frac{2\delta^2 \epsilon_H H^4}{4\pi^2}\delta(N - N') \approx 0, \\ \langle \tau(N)\sigma(N') \rangle &\approx \frac{\epsilon_H H^3}{4\pi^2}\delta(N - N')\end{aligned}$$

While both cases are very illustrative and somewhat similar (specifically, in the regime $\delta \ll 1$), here for our purposes we will focus on the first one, in which functional integrations are simplified greatly. The opposite case (b) is considered in relative depths in [40] and will be discussed in more details in a subsequent work.

To derive the Fokker-Planck equation and corrections to it, we follow the path integral approach outlined in the appendix D. It can be seen easily that the diffusion matrix associated with the correlation properties of the noises is singular in the quasi-de Sitter limit (a), and the functional integration measure for the noise terms has the form

$$Z_{\text{noise}} = \int \mathcal{D}\sigma \mathcal{D}\tau \exp\left(-\frac{1}{2} \int dN f^T D^{-1} f\right), \quad (4.13)$$

where

$$D = \begin{pmatrix} \frac{H^2(1+\delta^2)}{4\pi^2} & \frac{H^3\delta^2(1+i\delta)}{4\pi^2} \\ \frac{H^3\delta^2(1-i\delta)}{4\pi^2} & \frac{\delta^4 H^4}{4\pi^2} \end{pmatrix} \quad (4.14)$$

and $f^T = (\sigma, \tau)$. The matrix D is manifestly singular, and the noises σ and δ are correlated. Calculating eigenvectors and eigenvalues of the matrix D , we find that

$$\tau = -H\sigma \frac{i\delta^2}{i + \delta}, \quad (4.15)$$

while the non-trivial contribution into (4.13) is given by the combination $H^{-1}\tau + \frac{i(\delta-i)}{\delta^2}\sigma$. Since the matrix D is singular only in the limit of vanishing slow roll parameters $\epsilon_H \rightarrow 0$,

it should be kept in mind that its eigenvector corresponding to the zero eigenvalue really introduces a constraint on the dynamics of v and Φ .

The correlation functions of Φ and v can in turn be obtained by integrating over the measure

$$F = \int \mathcal{D}\sigma \mathcal{D}\tau \mathcal{D}v \mathcal{D}\Phi \mathcal{D}\lambda \mathcal{D}\mu \exp \left(\int dN \left(i\lambda \left(\frac{\partial v}{\partial N} + 3v + \frac{\partial V/\partial \Phi}{H} - \tau \right) + i\mu \left(\frac{\partial \Phi}{\partial N} - \frac{v}{H} - \sigma \right) \right) \right) Z_{\text{noise}}. \quad (4.16)$$

As all integrations (with the exception of the integration over Φ) are Gaussian, they can be explicitly taken revealing

$$F = \int \mathcal{D}\Phi \mathcal{D}v \exp(-S),$$

where

$$S = \int dN \mathcal{L} = \frac{1}{2} \int dN \frac{2\pi^2}{H^2(1+\delta^2+\delta^4)} \left(\frac{1+i\delta}{\delta^2} \left(\frac{\partial \Phi}{\partial N} - \frac{v}{H} \right) + \frac{1}{H} \left(\frac{\partial v}{\partial N} + 3v + \frac{1}{H} \frac{\partial V}{\partial \Phi} \right) \right)^2. \quad (4.17)$$

On top of this effective action there is a constraint present in the system (the one corresponding to the vanishing eigenvalue of the matrix (4.14)):

$$\frac{\delta^2}{1-i\delta} \left(\frac{\partial \Phi}{\partial N} - \frac{v}{H} \right) = \frac{1}{H} \left(\frac{\partial v}{\partial N} + 3v + \frac{1}{H} \frac{\partial V}{\partial \Phi} \right) \quad (4.18)$$

Solving it for $\frac{\partial \Phi}{\partial N} - \frac{v}{H}$ and substituting back into the action (4.17), we obtain the effective theory of the field v :

$$Z = \int \mathcal{D}v \mathcal{D}\Phi \exp \left(-\frac{2\pi^2 K}{H^4} \left(\frac{\partial v}{\partial N} + 3v + \frac{1}{H} \frac{\partial V}{\partial \Phi} \right)^2 \right), \quad (4.19)$$

where $K = (1+\delta^2)^2/(\delta^8 \cdot (1+\delta^2+\delta^4))$ (in this representation, Φ is considered an external field which we average out).

The conjugate momentum for the field v is given by

$$p_v = \frac{\pi^2 K}{H^4} \left(\frac{\partial v}{\partial N} + 3v + \frac{1}{H} \frac{\partial V}{\partial \Phi} \right) \quad (4.20)$$

and the Hamiltonian of the theory (4.19) is

$$\mathcal{H}_v = -\frac{H^4}{\pi^2 K} p_v^2 - 3v - \frac{1}{H} \frac{\partial V}{\partial \Phi}. \quad (4.21)$$

The Fokker-Planck equation for the probability $P(v, N)$ to measure a given value of v in a given Hubble patch is then obtained by writing down $\frac{\partial P(v, N)}{\partial N} = \mathcal{H}_v(p_v, v)P(v, N)$ and promoting conjugate momentum p_v into a differential operator according to the usual prescription $p_v = -\partial_v$, see appendix D.

An important conclusion is that the theory with the Hamiltonian (4.21) is generally unstable, with a run-away behavior of the probability $P(v, N)$. It can be shown that this conclusion survives in the general case, independent on relations between ϵ_H and δ (as we

shall demonstrate in the follow-up work): namely, the run-away behavior is associated with the behavior of the probability distribution P as a function of p_v , while its behavior as a function of p_Φ and Φ remains stable. This is a reflection of general instability of de Sitter space [52]. One and perhaps the only way to deal with this instability is to set a general constraint $p_v = 0$ (i.e., to choose initial conditions for the physical system in this rather special way). Then, from the constraint (4.18) we obtain

$$v = H \frac{\partial \Phi}{\partial N} \quad (4.22)$$

and substituting it back into the effective action of the theory (4.17), we finally obtain the theory with Lagrangian

$$\mathcal{L} = \frac{2\pi^2}{H^4(1 + \delta^2 + \delta^4)} \left(\frac{\partial}{\partial N} \left(H \frac{\partial \Phi}{\partial N} \right) + 3H \frac{\partial \Phi}{\partial N} + \frac{1}{H} \frac{\partial V}{\partial \Phi} \right)^2. \quad (4.23)$$

It is then straightforward to show that the theory (4.23) produces the canonical Starobinsky-Fokker-Planck equation with a singular correction $\sim A\delta(\Phi - \Phi')$ originating from the first term in parentheses in (4.23) and $A \sim H^2$.

5 Conclusion

Numerical and theoretical analysis of non-renormalizable field theories and 4-dimensional quantum gravity performed here shows that introducing a network of observers distributed in the world volume of the theory and continuously measuring scalar curvature leads to a drastic non-perturbative restructuring of the Hilbert space of the underlying theory significantly changing its infrared structure. Perhaps, most importantly, integrating out observers induces a de Sitter-like background of the theory. This in turn completely determines the infrared structure of correlation functions of physical observables. Moreover, introducing random ensembles of observers renormalizes the gravitational constant as well. Namely, while the magnitude of the cosmological constant is determined by the standard deviation of the observer coupling to the scalar curvature across the ensemble, the renormalized Planck mass squared is proportional to the average observer coupling to the scalar curvature, where the corresponding average and standard deviation are calculated using the distribution function of observer couplings. Thus, the induced cosmological constant and the gravitational constant are determined by the properties of observers — the distribution of observation events in the world volume of the theory and coupling strength between observers and gravitational degrees of freedom.

On the one hand, restructuring of the Hilbert space of the gravity coupled to observers is similar to the phenomenon of Anderson-like localization in disordered media. On the other hand, it is characterized by an effective dimensional reduction close in spirit to the celebrated Parisi-Sourlas dimensional reduction observed in several field theories in the presence of random external field (such as continuum limit of RFIM — random field Ising model).

In the case of gravity, this Hilbert space restructuring can be roughly characterized as follows. It is known by now that 4D simplicial Euclidean quantum gravity admits a UV fixed point at a particular value of $G_c = 1/(8\pi k_c)$, with a UV, strongly coupled phase at $k < k_c$ and an IR, weakly coupled phase, which is realized at $k > k_c$. The strongly coupled phase admits a 4-dimensional Euclidean Anti de Sitter like behavior of the ground state of gravity (i.e., expectation value of the scalar curvature remains negative in this phase) and a

non-trivial infrared dynamics of correlation functions of observables. On the other hand, the weakly coupled phase (which should be the one of physical interest as the real world gravity is weakly coupled!) seems to feature a quasi-2-dimensional branching polymer-like behavior without any smooth background geometry in the IR. This was previously interpreted as an absence of a proper continuum limit of the theory in this regime. Instead, we believe that this behavior is physical: in the weakly coupled regime the ground state of the theory admits a dS-like physics, and a 2-dimensional branching polymer, self-reproducing behavior of observables is really nothing but a Wick-rotated equivalent of the eternal inflation happening on this dS-like background.

Quenched disorder associated with random networks of observers measuring the strength of gravitational interaction clears up the mist somewhat in this respect: it seems to move the critical point of the theory towards $k_c \rightarrow 0$ implying that the only accessible phase of the theory is the one of weakly coupled gravity. Our theoretical analysis further shows that the nature of the effective dimensional reduction ($4D \rightarrow 2D$) in the presence of quenched disorder is associated with the fact that infrared dynamics of observables in the eternally inflating Universe is determined by the probability $P(\chi, \log(a)) = \exp(S(\chi, \log(a)))$ to measure a given value of the effective “inflaton” χ in a given Hubble patch. In other words, all correlation functions of physical observables are entirely determined by the structure of $P(\chi, \log(a))$. We find this observation rather interesting.

The discussed phenomenon might also explain what should really be understood by the “continuum limit” of non-renormalizable quantum gravity. Indeed, it is generally accepted that the formal continuum limit of non-renormalizable quantum field theories (including in principle 4-dimensional quantum general relativity, which might be non-perturbatively renormalizable) does not exist. Nevertheless, it is still possible to make a number of conclusions regarding the physical properties of such theories in the large-scale/infrared limit. To a degree, the way how to do it can be understood using the correspondence between relativistic quantum field theories and the corresponding statistical classical field theories obtained from the former by Wick rotation [53]. According to this correspondence, the classical statistical counterpart of a renormalizable quantum field theory describes behavior of the order parameter of a classical statistical system in a vicinity of a second order phase transition. At temperatures close to T_c the correlation length $\xi \sim |T - T_c|^{-\alpha}$ of the relevant degrees of freedom approaches infinity, which makes it possible to describe correlation functions of observables in terms of a small number of continuous order parameters only. Similarly, the statistical physics counterpart of a *non-renormalizable* quantum field theory describes a vicinity of a *first order* phase transition, when the correlation length ξ of physical degrees of freedom remains finite at all accessible values of thermodynamic potentials. The process of measuring the physical state of the field in an equivalent QFT can be thought of as an insertion of a projection operator in the world volume of the theory at a point of spacetime, where and when the measurement/observation is performed. In the statistical mechanical counterpart, such insertion is akin to an introduction of a heavy, “quenched” impurity in the spatial volume of the classical thermodynamic system, with elementary excitations of the order parameter(s) scattering against it. It can thus be expected that the network of von Neumann observers in a QFT is reminiscent of an ensemble of impurities introduced into the classical system described by a statistical counterpart of the theory. It is well known that in the vicinity of a first order phase transition, such impurities serve as nucleation centers for bubbles of the true phase [54]. When coupling constants of the von Neumann detectors to the field are sufficiently large, bubble nucleation process proceeds ad infinitum in a quasi-continuum limit

$T \rightarrow T_c$. Correspondingly, in a QFT, the vacuum state remains largely inhomogeneous even in the limit of Langevin time $\tau \rightarrow \infty$; the resulting state is also strongly dependent on the particular location of inserted operators describing observation events. Thus, the structure of the Hilbert space of a non-renormalizable quantum field theory is largely determined by “localization properties” of the effective potential of impurities inserted into an equivalent statistical mechanical system.

To conclude, two observations presented here point out towards a possible $4D \rightarrow 2D$ dimensional reduction of quantum gravity (at least in the infrared) in the presence of random networks of observers: (a) lattice simulations of simplicial Regge-Wheeler Euclidean gravity in the presence of disorder showing that after averaging over such disorder the critical exponent(s) of the theory change as if the effective dimensionality of the theory changes from $D = 1 + 3$ to $D = 1 + 1$, and (b) theoretical analysis of quantum (Lorentzian) general relativity in the presence of quenched disorder. Although these two separate approaches lead to the same conclusion regarding the physical system in question, ultimately only numerical simulations Lorentzian quantum gravity will allow to reconcile the two approaches. We hope to return to this subject in our future studies.

A Notes on von Neumann observers

Our notation of a physical observer is similar to the one used by Unruh [22] and DeWitt [23]. Namely, in field theoretic setups discussed below we assume that the detector is moving along a world line $x = x(\tau)$, where τ is its proper time, and interacting with the physical field as described by the monopole interaction term in the Lagrangian of the joint system “detector-field” $I(\tau)\phi(x(\tau))$ (note that the monopole coupling strength I of the detector to the field ϕ might itself depend on the proper time of the detector). In the case of gravity, we consider a similar interaction term coupling the monopole detector with the covariant term in the Einstein-Hilbert action, e.g., Ricci scalar, writing this interaction term as $I(\tau)R(x(\tau))$. Further, since we are interested in extremely long spatio-temporal scales (possible infrared limit of the theory), we assume that the lifetime of any particular detector, i.e., the proper total length of its trajectory $L = \int d\tau$ is much shorter than any physical scales of interest. Thus, world lines of monopole detectors can be essentially represented by delta-functions in the world volume of the theory. In the main text, we consider the situation when a large number of such detectors is placed in a world volume of the theory, characterized by varying strengths $I(x)$. The random ensemble of the monopole coupling strengths $I(x)$ of these detectors can be essentially considered as a single “frozen” realization of a scalar field, which we denote in the text as ϕ . Finally, we allow detectors to “consciously” exchange information about the measurements of the Ricci scalar between each other and can thus average over realizations of ϕ assuming ergodicity of their distribution in the spacetime (i.e., assuming that the average over realizations of the field ϕ is equivalent to the average over the 4-volume of the theory); in this sense, we denote these detectors as von Neumann-Wigner observers.

B Parisi-Sourlas dimensional reduction in non-renormalizable field theories in the presence of observer networks

Quantum gravity can be thought of as a “quantum field theory” with an infinite dimensional gauge symmetry (Lorentz groups of local coordinate transformations in every point of spacetime) [2, 3]). It naturally makes sense to consider simplified models of the same phenomenon

which we described above by making dimensionality of the symmetry group finite and recall how Parisi-Sourlas dimensional reduction (due to the presence of random observer networks) emerges in the class of theories with finite-dimensional symmetries.

The first analyzed model of interest is a non-renormalizable scalar field theory with Z_2 global symmetry in $D = 5$ and 6 spacetime dimensions, with the Lagrangian density of the form $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda_0\phi^4 - \dots$, where $\dots$ denotes higher-order terms in powers of the scalar field ϕ and its spacetime derivatives ∂ . It is well-known that this theory is trivial [48, 55, 56], which implies that all its critical exponents coincide with the ones given by the mean field theory approximation (with logarithmic corrections in 4 dimensions [56]). In other words, if the number of spacetime dimensions $D > 4$, the quantum effective action of the ϕ^4 theory in the continuum limit can be well described by the one of a free massive scalar field theory with an effective mass dependent on the bare coupling λ_0 .

Consider a system of von Neumann detectors excited during the interaction events with quanta of the field ϕ [57, 58]. As usual, such detectors with monopole moments $J_i = J_i(t, x)$ can be modeled by terms in the Lagrangian density of the theory linear in the field variable ϕ as

$$Z = \int \mathcal{D}\phi \exp \left(-i \sum_j \int d^D x \left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_0^2\phi^2 - \frac{1}{4}\lambda_0\phi^4 + J_j\phi \right) \right).$$

Here the sum $\sum_i$ runs over detectors distributed in the world volume of the theory. It is often convenient to think of sources J_i as a second, extra massive scalar field (with a suppressed kinetic term). We are specifically interested in the case when a very large number of such von Neumann detectors randomly located in the world volume of the theory is present. The detectors are characterized by random couplings J_i to the field ϕ .

It can be seen straightforwardly that the physical setup described here is equivalent to the one realized in a quantum $\lambda\phi^4$ theory in a random external field or its discrete version, the random field D -dimensional Ising model (RFIM) well studied in literature, see for example [59–62]). A celebrated result by Parisi and Sourlas [16] states that the infrared behavior of RFIM is equivalent to the one of a similar theory (Ising model) in the absence of random external field but living in $(D - 2)$ dimensions. Namely, the most infrared divergent terms present in the perturbative expansion of the generating functionals of the two theories ($(D - 2)$ -dimensional IM and D -dimensional RFIM) coincide term by term. While the Parisi-Sourlas correspondence for Ising model breaks down for $D < 3$ [60], it has been shown to hold universally for $D \geq 4$.

Of especial interest for us is the observation that the presence of a large number of von Neumann detectors drastically changes the structure of the Hilbert space of the theory. For $D = 4$ and 5 the Hilbert space can no longer be approximated by the mean field-theoretic partition function once the disorder associated with the external field is averaged out. This is reflected in the change of critical exponents of the theory as well as correlation functions of the field.

The magnitude of the change in the structure of Hilbert space of the theory can be assessed by numerical simulations. Those have been recently done in [60] for $D = 3$ random field Ising model (RFIM), in [59, 61] for $D = 4$ RFIM and in [62] for $D = 5$ RFIM. We have also performed lattice numerical simulations of RFIM and reproduced the known results for comparison of $D = 4, 5$ RFIM with pure Ising model in $D = 2, 3$ dimensions. Similar to [59–61] we have exploited the fact that RFIM achieves phase transition at zero temperature [63]

and as such, it is sufficient to focus on the physics of the ground state of the theory. For numerical simulations, we have used minimum cost-flow algorithm [64, 65].

In addition, we have also performed simulations of $D = 6$ RFIM and compared its behavior with the one of pure Ising model in $D = 4$ dimensions. As was expected, Parisi-Sourlas dimensional reduction was observed in 5-dimensional and 6-dimensional RFIM, with the critical exponents of $D = 4$ RFIM deviating from the ones of 2-dimensional Ising model due to the known breakdown of dimensional reduction mechanism in lower dimensions. Estimating critical exponents for $D = 6$ RFIM we were unable to detect logarithmically weak corrections to the mean field approximation.

To confirm universality of Hilbert space restructuring in quantum field theories due to the presence of networks of observers/observation events, we have also performed numerical simulations of Z_2 gauge theories. Those included theories in $D = 2 + 1$ and $3 + 1$ spacetime dimensions [66–68], $D = 4 + 1$ and $5 + 1$ spacetime dimensions (here $+1$ denotes the dimension with periodic boundary conditions). Von Neumann observers were modeled by a scalar degree of freedom coupled to the Z_2 gauge field with the resulting free energy of the theory given by

$$F = \sum_{i,j,k,l} \sigma_{ij} \sigma_{jk} \sigma_{kl} \sigma_{li} + \sum_{i,j,n} g_n \tau_i \sigma_{ij} \tau_j, \quad (\text{B.1})$$

where the couplings g_n (strengths of detectors' couplings to Z_2 gauge field) and locations of insertions of the quenched disorder elements (observation events) were considered random and Gaussian-distributed. Again, we have observed effective dimensional reduction in the random field Z_2 gauge theory implying universality of this phenomenon across a wide range of theories with global and gauge symmetries.

C Numerical simulations of field theories

C.1 Lattice simulations of Ising model and random field Ising model (RFIM)

The Ising model approximates (Euclidean) ϕ^4 quantum field theory in the continuum limit (achieved for RFIM at zero temperature [63]). Lattice simulations of zero-temperature RFIM in $D = 4, 5$ and 6 dimensions were performed on hypercubic lattices with sizes $L = 8, 10, 12, 16$ and 20 . Ground states of the resulting IMs were calculated for 10^6 realizations of disorder. For both IM and RFIM, finite-size scaling effects were taken into account. After extraction of L -dependence, the values of critical exponents were determined by extrapolating $L^{-1} \rightarrow 0$. We obtained $\eta = 0.1942 \pm 0.0022$, $\nu = 0.8726 \pm 0.0182$ for $D = 4$, $\eta = 0.0442 \pm 0.0032$, $\nu = 0.6293 \pm 0.0030$ for $D = 5$ and $\eta = 0.0103 \pm 0.0041$, $\nu = 0.4892 \pm 0.0171$ for $D = 6$.

C.2 Lattice simulations of pure and random field Z_2 gauge field theories

Monte-Carlo lattice simulations of Euclidean Z_2 and RF (random field) Z_2 gauge field theories were performed on periodic hypercubic lattices of the size $L = 8, 12, 18, 24$ and 28 for the spatial part and fixed $L = 2$ for the inverse temperature part of the lattice. For the RF Z_2 gauge field theory, 10^6 realizations of random disorder were used. For Z_2 gauge field theory, we obtained $\beta = 0.13 \pm 0.02$, $\nu = 0.99 \pm 0.03$ for $D = 2 + 1$ and $\beta = 0.33 \pm 0.01$, $\nu = 0.63 \pm 0.03$ for $D = 3 + 1$ dimensions. (As usual, $+1$ denotes a dimension with periodic Matsubara boundary conditions.) For RF Z_2 gauge field theory, we found $\beta = 0.11 \pm 0.03$, $\nu = 0.65 \pm 0.04$ for $D = 4 + 1$ and $\beta = 0.30 \pm 0.05$, $\nu = 0.65 \pm 0.04$ for $D = 5 + 1$ dimensions.

D Deriving Starobinsky-Fokker-Planck equation using path integral approach

In this appendix we shall illustrate the power of path integral approach for analyzing infrared dynamics of the scalar field in quasi-de Sitter universe by deriving the standard inflationary Fokker-Planck equation in the one-noise model. As usual we start with

$$\frac{\partial \Phi}{\partial N} = -\frac{1}{3H^2} \frac{\partial V}{\partial \Phi} + \frac{f}{H}, \quad (\text{D.1})$$

where the noise $f = H\sigma$ possesses the correlation properties

$$\langle \sigma(N) \sigma(N') \rangle = \frac{H^2}{4\pi^2} \delta(N - N'). \quad (\text{D.2})$$

This equation is derived straightforwardly using the approach described in [38] under the assumption of vanishing slow roll parameters $\epsilon_H, \eta_H \rightarrow 0$. The partition function of the effective IR theory thus has the form

$$Z = \int \mathcal{D}\Phi \mathcal{D}\sigma \delta \left(\frac{\partial \Phi}{\partial N} + \frac{1}{3H^2} \frac{\partial V}{\partial \Phi} - \sigma \right) \exp \left(- \int dN \frac{2\pi^2}{H^2} \sigma^2 \right).$$

Introducing a Lagrangian multiplier for the functional delta function, integrating out the noise σ as well as the Lagrangian multiplier, we obtain

$$Z = \int \mathcal{D}\Phi \exp \left(- \int dN \frac{2\pi^2}{H^2} \left(\frac{\partial \Phi}{\partial N} + \frac{1}{3H^2} \frac{\partial V}{\partial \Phi} \right)^2 \right). \quad (\text{D.3})$$

Thus the Lagrangian of the theory is

$$\mathcal{L} = \frac{2\pi^2}{H^2} \left(\frac{\partial \Phi}{\partial N} + \frac{1}{3H^2} \frac{\partial V}{\partial \Phi} \right)^2. \quad (\text{D.4})$$

The momentum conjugate to Φ is given by

$$P_\Phi = \frac{\partial \mathcal{L}}{\partial \Phi'} = \frac{\pi^2}{H^2} \left(\frac{\partial \Phi}{\partial N} + \frac{1}{3H^2} \frac{\partial V}{\partial \Phi} \right), \quad (\text{D.5})$$

and the Hamiltonian of the theory corresponding to the Lagrangian (D.4) is

$$H_\Phi = P_\Phi \Phi' - \mathcal{L} = P_\Phi \Phi' - \mathcal{L} = \frac{1}{8\pi^2} P_\Phi H^2 P_\Phi - \frac{P_\Phi}{3H^2} \frac{\partial V}{\partial \Phi}. \quad (\text{D.6})$$

The Starobinsky-Fokker-Planck equation [38] describing IR inflationary dynamics is obtained using this Hamiltonian and replacing $P_\Phi \rightarrow -\frac{\partial}{\partial \Phi}$ in the same fashion as Schroedinger equation is derived from the Feynman path integral for quantum mechanics:

$$\frac{\partial P(\Phi, N)}{\partial N} = H_\Phi \left(-\frac{\partial}{\partial \Phi}, \Phi \right) P(\Phi, N). \quad (\text{D.7})$$

E Non-locality in the one-noise model

E.1 Useful preliminary expressions and used notations

E.1.1 Slow roll parameters

In what follows, we consider the case of a single scalar field with a potential $V(\phi)$ propagating in a FRW spacetime with metric $ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 = a^2(t)(d\eta^2 - d\mathbf{x}^2)$. Ignoring dependence on spatial coordinates $\mathbf{x}$, slow roll parameters are defined according to the usual prescription

$$\epsilon_H = \frac{M_P^2}{4\pi} \left(\frac{dH/d\phi}{H} \right)^2, \quad (\text{E.1})$$

$$\eta_H = \frac{M_P^2}{4\pi} \frac{d^2 H/d\phi^2}{H}. \quad (\text{E.2})$$

Using the Hamilton-Jacobi equation for inflation

$$\left(\frac{dH}{d\phi} \right)^2 - \frac{12}{M_P^2} H^2 = \frac{32\pi^2}{M_P^4} V(\phi) \quad (\text{E.3})$$

and expressions

$$\frac{d\phi}{dt} = -\frac{M_P^2}{4\pi} \frac{dH}{d\phi}, \quad \frac{dH}{dt} = -\frac{4\pi}{M_P^2} \left(\frac{d\phi}{dt} \right)^2 \quad (\text{E.4})$$

one can demonstrate that

$$\epsilon_H = -\frac{dH/dt}{H^2}. \quad (\text{E.5})$$

Using slow roll parameters ϵ_H, η_H rather than the usual slow roll parameters ϵ_V, η_V is more convenient as the end of inflationary stage corresponds to the condition $\epsilon_H = 1$ being held exactly.

Other useful formulae which we use below include

$$\begin{aligned} \frac{\partial(aH)}{\partial\eta} &= (aH)^2(1 - \epsilon_H), \\ \frac{\partial\epsilon_H}{\partial\eta} &= -2(\epsilon_H - \eta_H)\epsilon_H aH, \\ \frac{\partial^2(aH)}{\partial\eta^2} &= (1 - 2\epsilon_H + 2\epsilon_H^2 - \epsilon_H\eta_H)(aH)^3. \end{aligned} \quad (\text{E.6})$$

The formula (E.6) rewritten in terms of inflationary efoldings $dN = H a d\eta$ shows that taking time derivatives of slow roll parameters produces terms of higher order in slow roll expansion.

E.1.2 Number of inflationary efoldings

The Langevin and Fokker-Planck equations derived below are written in terms of the number of efoldings $N = \log a$ rather than the world time t or conformal time η . It is therefore appropriate to introduce the Jacobians associated with the corresponding change of variables. We find:

$$\frac{\partial}{\partial\eta} = aH \frac{\partial}{\partial N}.$$

A number of useful formulae which will be used in later derivations follow

$$\begin{aligned}\epsilon_H &= -\frac{1}{H} \frac{dH}{dN}, \quad \eta_H = \epsilon_H - \frac{M_P}{\sqrt{16\pi}} \frac{d\epsilon_H/d\phi}{\sqrt{\epsilon_H}}, \\ \frac{a''}{a} &= (2 - \epsilon_H)(Ha)^2, \\ \frac{\partial}{\partial N}(aH) &= aH(1 - \epsilon_H).\end{aligned}$$

E.2 Separating scalar field into IR and UV parts. Langevin equation

The separation of the field into subhorizon and superhorizon parts is done according to

$$\phi = \Phi + \frac{1}{(2\pi)^{3/2}} \int d^3k \theta(k - \delta aH) \left(a_k \phi_k(\eta) e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.} \right). \quad (\text{E.7})$$

Here as usual $\theta(\dots)$ is the Heaviside step function of the argument and δ is a free dimensionless parameter identified with IR/UV separation scale (usually, in stochastic formalism it is taken to be small, $\delta \ll 1$, but not too small in order for potential terms to remain sub-dominant). Substituting the expression into the operator equation $\square\phi + \frac{\partial V}{\partial\phi} = 0$ and neglecting potential term for the UV part of the field, we obtain:

$$\square\Phi + \frac{1}{a^3}(u'' - \nabla^2 u - \frac{a''}{a}u + \dots) \approx -\frac{\partial V}{\partial\Phi}, \quad (\text{E.8})$$

where $\dots$ denotes terms related to the potential and

$$\square\Phi = H^2 \frac{\partial^2 \Phi}{\partial N^2} + H^2(3 - \epsilon_H) \frac{\partial \Phi}{\partial N} - \frac{\nabla^2 \Phi}{a^2}$$

and

$$u = \frac{a}{(2\pi)^{3/2}} \int d^3k \theta(k - \delta aH) \left(a_k u_k(\eta) e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.} \right).$$

Substituting u into the eq. (E.8) and using the expression

$$\begin{aligned}u'' &= -\frac{\delta^2}{(2\pi)^{3/2}} \int d^3k \delta'(k - \delta aH) (aH)^4 (1 - \epsilon_H)^2 (a_k u_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}) - \\ &\quad - \frac{2\delta}{(2\pi)^{3/2}} \int d^3k \delta(k - \delta aH) (aH)^3 (1 - 2\epsilon_H + 2\epsilon_H^2 - \epsilon_H \eta_H) (a_k u_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}) - \\ &\quad - \frac{2\delta}{(2\pi)^{3/2}} \int d^3k \delta(k - \delta aH) (aH)^2 (1 - \epsilon_H) (a_k u'_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}) + \\ &\quad + \frac{1}{(2\pi)^{3/2}} \int d^3k \theta(k - \delta aH) (a_k u''_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}),\end{aligned}$$

we finally obtain the “Langevin” equation for the infrared part of the field

$$\frac{\partial^2 \Phi}{\partial N^2} + (3 - \epsilon_H) \frac{\partial \Phi}{\partial N} - \frac{\nabla^2 \Phi}{(aH)^2} + \frac{\partial^2 V}{\partial \Phi^2} = f_1(t, \mathbf{x}) + f_2(t, \mathbf{x}), \quad (\text{E.9})$$

where the “noise” operators f_1 and f_2 are defined according to

$$f_1 = -\frac{\delta^2 a H^2 (1 - \epsilon_H)^2}{(2\pi)^{3/2}} \int d^3k \delta'(k - \delta aH) (a_k u_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}), \quad (\text{E.10})$$

$$\begin{aligned}f_2 &= -\frac{2\delta}{(2\pi)^{3/2}} \int d^3k \delta(k - \delta aH) \left[H(1 - 2\epsilon_H + 2\epsilon_H^2 - \epsilon_H \eta_H) (a_k u_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}) + \right. \\ &\quad \left. \frac{1 - \epsilon_H}{a} (a_k u'_k e^{-i\mathbf{k}\mathbf{x}} + \text{h.c.}) \right]. \quad (\text{E.11})\end{aligned}$$

The part of the noise term f_1 seems to be suppressed at small $\delta \ll 1$. However, as we shall see, generally this is not the case as powers of δ are canceled out in the observable quantities.

E.3 Commutation relations of the noise operators f_1 and f_2

While the noise terms commute with terms on the r.h.s. of the eq. (E.9), it is also useful to check their self-commutation relations. Due to ultra-locality in the quasi-de Sitter regime, we shall be particularly interested in commutation relations of operators at the same spatial point $\mathbf{x}$. We find that

$$[f_1(N), f_1(N')] = 0, \quad (\text{E.12})$$

$$[f_2(N), f_2(N')] = 0, \quad (\text{E.13})$$

$$[f_1(N), f_2(N')] = -\frac{i\delta^3 H^2 (1 - \epsilon_H)}{\pi^2} \delta'(N - N'). \quad (\text{E.14})$$

It is worth noting that although the Langevin equation (E.9) is considered to be quasi-classical, operators f_1 and f_2 are not generally commuting although their commutator is small at $\delta \ll 1$ and becomes vanishing by the end of inflation when $\epsilon_H \rightarrow 1$.

This is in contrast with the Langevin-Starobinsky equation [38] for a scalar field on a fixed de Sitter background

$$\dot{\Phi} + \frac{1}{3H_0} \frac{\partial V}{\partial \Phi} = f, \quad (\text{E.15})$$

where

$$f = -\frac{\delta a H_0^3 i}{4\pi^{3/2}} \int d^3 k \delta(k - \delta a H_0) \frac{1}{k^{3/2}} \left(a_k e^{-i\mathbf{k}\mathbf{x}} - a_k^\dagger e^{i\mathbf{k}\mathbf{x}} \right). \quad (\text{E.16})$$

One can immediately see that due to antisymmetric form of the combination $(a_k e^{-i\mathbf{k}\mathbf{x}} - a_k^\dagger e^{i\mathbf{k}\mathbf{x}})$ the noise term $f(t)$ commutes with itself if the same spatial point is considered:

$$[f(t, \mathbf{x}), f(t', \mathbf{x})] = 0.$$

For points with large spatial (superhorizon) separation the noise terms do not commute even in this simplified case:

$$[f(t, \mathbf{x}), f(t', \mathbf{x}')] = \frac{H^3}{2\pi^2} \sin(\epsilon a H |\mathbf{x} - \mathbf{x}'|) \delta(t - t').$$

The reason of this discrepancy with our result is due to dropping terms $\sim f_1$ as suppressed by additional powers of δ at $\delta \ll 1$ during the derivation of (E.16); on the other hand, when deriving (E.12)–(E.14) all terms are kept explicitly. It is thus useful to remember that the quantum nature of the noise in the Langevin equation (E.9) is not eradicated completely during the quasi-de Sitter inflationary stage when $\epsilon_H \ll 1$.

E.4 Correlation functions of the noise operators f_1 and f_2

While we consider the field operators $f_1(N, \mathbf{x})$ and $f_2(N, \mathbf{x})$ quasi-classical quantities (based on their commutation relations in the regime $\delta \ll 1$ as well as their commuting with other terms in the Langevin equation (E.9)) in what follows, it is necessary to determine their

stochastic properties. Those are given by the expectation values in the vacuum state of the Fock space of modes u_k . We find:

$$\begin{aligned} \langle f_2(N, \mathbf{x}) f_2(N', \mathbf{x}) \rangle &= \frac{2\delta^2 H}{\pi^2} \left[\left(\frac{1 - 2\epsilon_H + 2\epsilon_H^2 - \epsilon_H \eta_H}{\sqrt{1 - \epsilon_H}} \text{Re} u_k + \frac{\sqrt{1 - \epsilon_H}}{Ha} \text{Re} u'_k \right)^2 + \right. \\ &\quad \left. + \left(\frac{1 - 2\epsilon_H + 2\epsilon_H^2 - \epsilon_H \eta_H}{\sqrt{1 - \epsilon_H}} \text{Im} u_k + \frac{\sqrt{1 - \epsilon_H}}{Ha} \text{Im} u'_k \right)^2 \right]_{k=\delta a H} \delta(N - N'), \end{aligned} \quad (\text{E.17})$$

$$\begin{aligned} \langle f_1(N, \mathbf{x}) f_1(N', \mathbf{x}) \rangle &= -\frac{H^2 \delta^2}{2\pi^2} \left[(1 - \epsilon_H) \delta a H (u_k u_k^*)_{k=\delta a H} \delta''(N - N') + \right. \\ &\quad + 2(1 - \epsilon_H)^2 \delta a H (u_k u_k^*)_{k=\delta a H} \delta'(N - N') + \\ &\quad \left. + (1 - \epsilon_H)^2 (\delta a H)^2 \left(\frac{d}{dk} (u_k u_k^*) \right)_{k=\delta a H} \delta'(N - N') \right], \end{aligned} \quad (\text{E.18})$$

$$\begin{aligned} \langle f_1(N, \mathbf{x}) f_2(N', \mathbf{x}) \rangle &= \frac{\delta^3 H^2}{\pi^2} \left[a H (1 - 2\epsilon_H + 2\epsilon_H^2 - \epsilon_H \eta_H) (u_k u_k^*)_{k=\delta a H} + \right. \\ &\quad \left. + (1 - \epsilon_H) (u_k u_k^*)_{k=\delta a H} \right] \delta'(N - N'). \end{aligned} \quad (\text{E.19})$$

We emphasize that the expressions (E.17)–(E.19) are exact to all orders in slow roll parameters ϵ_H , η_H . They can be significantly simplified if the leading order in slow roll parameters is kept; in the regime $\eta_H \rightarrow 0$, $\epsilon_H \rightarrow 0$ we have

$$\begin{aligned} u_k &\approx \frac{1}{\sqrt{2k}} \left(\frac{i}{k\eta} - 1 \right) e^{-ik\eta} = -\frac{1}{\sqrt{2k}} \left(\frac{iHa}{k} + 1 \right) e^{\frac{ik}{Ha}}, \\ u'_k &\approx \frac{1}{\sqrt{2k\eta}} e^{-ik\eta} \left(-\frac{i}{k\eta} - ik\eta + 1 \right) = -\frac{Ha}{\sqrt{2k}} e^{\frac{ik}{Ha}} \left(\frac{iHa}{k} + \frac{ik}{Ha} + 1 \right), \end{aligned}$$

and

$$\langle f_2(N, \mathbf{x}) f_2(N', \mathbf{x}) \rangle = \frac{4H^2}{\pi^2} \delta(N - N'), \quad (\text{E.20})$$

$$\langle f_1(N, \mathbf{x}) f_1(N', \mathbf{x}) \rangle = \frac{H^2}{8\pi^2} \delta'(N - N') - \frac{H^2}{4\pi^2} \delta''(N - N'), \quad (\text{E.21})$$

$$\langle f_1(N, \mathbf{x}) f_2(N', \mathbf{x}) \rangle = \frac{H^2}{\pi^2} \delta'(N - N'). \quad (\text{E.22})$$

We are thus forced to conclude that in the general case the one noise model produces a *non-local* effective theory, and a rather hard one to deal with. This can immediately be seen from the behavior of the correlation functions (E.20)–(E.22) as well as after integrating the noise $f_1 f_2$ out in the partition function of the theory. The non-locality cannot be really neglected as the correlation functions of the noise $\delta'(N - N')$, $\delta''(N - N')$ are not suppressed. It also hints on the presence of an additional stochastic degree of freedom which was integrated out to produce the non-local behavior and forces us to apply the two-noise model described in the main text.

E.4.1 Relation to the Starobinsky's stochastic formalism

Two observations are in order. First of all, we note that even keeping f_2 only (which is essentially equivalent to using the approximation $\epsilon_H \ll 1$, $\delta \ll 1$ employed in [38] and

throughout the literature) we do not reproduce Starobinsky’s result for the numerical factor in front of the correlation function (E.20). Namely, the difference between the two results is a factor of 3/4, which is crucial given that it determines the correct value for de Sitter entropy. To understand what happens, let us recall how it is derived. If all terms suppressed by higher powers of slow roll parameters are neglected, the resulting equation for the superhorizon part of the field has the form

$$\frac{\partial \Phi}{\partial t} = -\frac{1}{3H} \frac{\partial V}{\partial \Phi} + f,$$

where

$$f = -\frac{i\delta a H^3}{4\pi^{3/2}} \int d^3k \delta(k - \delta a H) \frac{1}{k^{3/2}} \left(a_k e^{-i\mathbf{k}\mathbf{x}} - a_k^\dagger e^{-i\mathbf{k}\mathbf{x}} \right).$$

Note however that the terms $\ddot{\phi}$ which we neglected in the equations above would also contain the contribution $\sim \dot{f}$. While most contributions to $\dot{f}$ are suppressed by additional powers of δ , there is also a contribution present which is proportional to $\sim H f$. This contribution is exactly the one which accounts for the difference between our and Starobinsky’s result. However, keeping terms like this, we should be extra careful since f is a stochastic variable, which we are trying to differentiate.

Second, we note that the correlators E.19 and E.18 are *not* suppressed by powers of δ and thus should generally be kept. The resulting theory (after the noise term f is integrated out) is non-local in N . This non-locality hints on an existence of an additional effective field variable which has been integrated out to obtain the resulting non-local theory.

Acknowledgments

The work of A.O.B. was supported by the RFBR grant No.20-02-00297 and by the Foundation for Theoretical Physics Development “Basis”.

References

- [1] A. Einstein, B. Podolsky and N. Rosen, *Can quantum mechanical description of physical reality be considered complete?*, *Phys. Rev.* **47** (1935) 777 [INSPIRE].
- [2] G. ’t Hooft and M.J.G. Veltman, *One loop divergencies in the theory of gravitation*, *Ann. Inst. H. Poincaré Phys. Theor.* **A20** (1974) 69.
- [3] S. Deser and P. van Nieuwenhuizen, *One Loop Divergences of Quantized Einstein-Maxwell Fields*, *Phys. Rev. D* **10** (1974) 401 [INSPIRE].
- [4] S.N. Solodukhin, *Entanglement entropy of black holes*, *Living Rev. Rel.* **14** (2011) 8 [arXiv:1104.3712] [INSPIRE].
- [5] S.W. Hawking, *Breakdown of Predictability in Gravitational Collapse*, *Phys. Rev. D* **14** (1976) 2460 [INSPIRE].
- [6] S. Weinberg, *The Cosmological Constant Problem*, *Rev. Mod. Phys.* **61** (1989) 1 [INSPIRE].
- [7] S. Weinberg, *Critical phenomena for field theorists*, in *Proceedings of the International School of Subnuclear Physics, Ettore Majorana Center for scientific culture*, Erice, Italy, 24–26 July 1976, pp. 1–52.
- [8] H.W. Hamber, *Quantum Gravity on the Lattice*, *Gen. Rel. Grav.* **41** (2009) 817 [arXiv:0901.0964] [INSPIRE].
- [9] J. Ambjørn and J. Jurkiewicz, *Four-dimensional simplicial quantum gravity*, *Phys. Lett. B* **278** (1992) 42 [INSPIRE].

- [10] S. Catterall, J.B. Kogut and R. Renken, *Phase structure of four-dimensional simplicial quantum gravity*, *Phys. Lett. B* **328** (1994) 277 [[hep-lat/9401026](#)] [[INSPIRE](#)].
- [11] P. Bialas, Z. Burda, A. Krzywicki and B. Petersson, *Focusing on the fixed point of 4 – D simplicial gravity*, *Nucl. Phys. B* **472** (1996) 293 [[hep-lat/9601024](#)] [[INSPIRE](#)].
- [12] M. Reuter, *Nonperturbative evolution equation for quantum gravity*, *Phys. Rev. D* **57** (1998) 971 [[hep-th/9605030](#)] [[INSPIRE](#)].
- [13] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The String landscape, black holes and gravity as the weakest force*, *JHEP* **06** (2007) 060 [[hep-th/0601001](#)] [[INSPIRE](#)].
- [14] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett. B* **379** (1996) 99 [[hep-th/9601029](#)] [[INSPIRE](#)].
- [15] J. Kaplan and S. Kundu, *Closed Strings and Weak Gravity from Higher-Spin Causality*, *JHEP* **02** (2021) 145 [[arXiv:2008.05477](#)] [[INSPIRE](#)].
- [16] G. Parisi and N. Sourlas, *Random Magnetic Fields, Supersymmetry and Negative Dimensions*, *Phys. Rev. Lett.* **43** (1979) 744 [[INSPIRE](#)].
- [17] A.M. Polyakov, *Quantum Geometry of Bosonic Strings*, *Phys. Lett. B* **103** (1981) 207 [[INSPIRE](#)].
- [18] V.G. Knizhnik, A.M. Polyakov and A.B. Zamolodchikov, *Fractal Structure of 2D Quantum Gravity*, *Mod. Phys. Lett. A* **3** (1988) 819 [[INSPIRE](#)].
- [19] T. Regge, *General relativity without coordinates*, *Nuovo Cim.* **19** (1961) 558 [[INSPIRE](#)].
- [20] J.A. Wheeler, *Geometrodynamics and the Issue of the Final State*, in *Relativity Groups and Topology*, Les Houches Lecture Notes (1963).
- [21] H.W. Hamber, *On the gravitational scaling dimensions*, *Phys. Rev. D* **61** (2000) 124008 [[hep-th/9912246](#)] [[INSPIRE](#)].
- [22] W.G. Unruh, *Notes on black hole evaporation*, *Phys. Rev. D* **14** (1976) 870 [[INSPIRE](#)].
- [23] B. DeWitt, *Quantum gravity, the new synthesis*, in *General Relativity: An Einstein centenary survey*, S.W. Hawking and W. Israel eds., Cambridge University Press (1979), pp. 680–745.
- [24] S. Weinberg, *Ultraviolet divergences in quantum theories of gravitation*, in *General Relativity: An Einstein centenary survey*, S.W. Hawking and W. Israel eds., Cambridge University Press (1979), pp. 790–832.
- [25] H. Kawai and M. Ninomiya, *Renormalization Group and Quantum Gravity*, *Nucl. Phys. B* **336** (1990) 115 [[INSPIRE](#)].
- [26] T. Aida, Y. Kitazawa, J. Nishimura and A. Tsuchiya, *Two loop renormalization in quantum gravity near two-dimensions*, *Nucl. Phys. B* **444** (1995) 353 [[hep-th/9501056](#)] [[INSPIRE](#)].
- [27] H.W. Hamber and R.M. Williams, *Higher derivative quantum gravity on a simplicial lattice*, *Nucl. Phys. B* **248** (1984) 392 [*Erratum* *ibid.* **260** (1985) 747] [[INSPIRE](#)].
- [28] H.W. Hamber and R.M. Williams, *Nonperturbative simplicial quantum gravity*, *Phys. Lett. B* **157** (1985) 368 [[INSPIRE](#)].
- [29] H.W. Hamber and R.M. Williams, *Gauge invariance in simplicial gravity*, *Nucl. Phys. B* **487** (1997) 345 [[hep-th/9607153](#)] [[INSPIRE](#)].
- [30] H.W. Hamber and R.M. Williams, *Non-perturbative gravity and the spin of the lattice graviton*, *Phys. Rev. D* **70** (2004) 124007 [[hep-th/0407039](#)] [[INSPIRE](#)].
- [31] B. Berg, *Exploratory numerical study of discrete quantum gravity*, *Phys. Rev. Lett.* **55** (1985) 904 [[INSPIRE](#)].

- [32] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, *Phys. Lett. B* **91** (1980) 99 [INSPIRE].
- [33] J.D. Barrow and S. Cotsakis, *Inflation and the Conformal Structure of Higher Order Gravity Theories*, *Phys. Lett. B* **214** (1988) 515 [INSPIRE].
- [34] A. De Felice and S. Tsujikawa, *$f(R)$ theories*, *Living Rev. Rel.* **13** (2010) 3 [arXiv:1002.4928] [INSPIRE].
- [35] A.O. Barvinsky and A.Y. Kamenshchik, *Quantum scale of inflation and particle physics of the early universe*, *Phys. Lett. B* **332** (1994) 270 [gr-qc/9404062] [INSPIRE].
- [36] A.O. Barvinsky, A.Y. Kamenshchik and A.A. Starobinsky, *Inflation scenario via the Standard Model Higgs boson and LHC*, *JCAP* **11** (2008) 021 [arXiv:0809.2104] [INSPIRE].
- [37] F.L. Bezrukov and M. Shaposhnikov, *The Standard Model Higgs boson as the inflaton*, *Phys. Lett. B* **659** (2008) 703 [arXiv:0710.3755] [INSPIRE].
- [38] A.A. Starobinsky, *Stochastic de sitter (inflationary) stage in the early universe*, in *Field theory, quantum gravity and strings*, Berlin, Germany: Springer (1988), pp. 107–126.
- [39] M. Sasaki, Y. Nambu and K.-i. Nakao, *Classical Behavior of a Scalar Field in the Inflationary Universe*, *Nucl. Phys. B* **308** (1988) 868 [INSPIRE].
- [40] Y. Nambu and M. Sasaki, *Stochastic Approach to Chaotic Inflation and the Distribution of Universes*, *Phys. Lett. B* **219** (1989) 240 [INSPIRE].
- [41] A.A. Starobinsky and J. Yokoyama, *Equilibrium state of a selfinteracting scalar field in the de Sitter background*, *Phys. Rev. D* **50** (1994) 6357 [astro-ph/9407016] [INSPIRE].
- [42] S.-J. Rey, *Dynamics of Inflationary Phase Transition*, *Nucl. Phys. B* **284** (1987) 706 [INSPIRE].
- [43] A. Hosoya, M. Morikawa and K. Nakayama, *Stochastic Dynamics of Scalar Field in the Inflationary Universe*, *Int. J. Mod. Phys. A* **4** (1989) 2613 [INSPIRE].
- [44] F.R. Graziani, *Quantum Probability Distributions in the Early Universe. 1. Equilibrium Properties of the Wigner Equation*, *Phys. Rev. D* **38** (1988) 1122 [INSPIRE].
- [45] I.D. Lawrie, *Perturbative Description of Dissipation in Nonequilibrium Field Theory*, *Phys. Rev. D* **40** (1989) 3330 [INSPIRE].
- [46] V. Castellani, S. Degl’Innocenti, P.G. Prada Moroni and V. Tordiglione, *Hipparcos open clusters and stellar evolution*, *Mon. Not. Roy. Astron. Soc.* **334** (2002) 193 [astro-ph/0203327] [INSPIRE].
- [47] K. Enqvist, S. Nurmi, D. Podolsky and G.I. Rigopoulos, *On the divergences of inflationary superhorizon perturbations*, *JCAP* **04** (2008) 025 [arXiv:0802.0395] [INSPIRE].
- [48] D.I. Podolsky, *On Triviality of $\lambda\phi^4$ Quantum Field Theory in Four Dimensions*, arXiv:1003.3670 [INSPIRE].
- [49] D. Podolskiy and R. Lanza, *On decoherence in quantum gravity*, *Annalen Phys.* **528** (2016) 663 [arXiv:1508.05377] [INSPIRE].
- [50] D. Podolskiy, *Microscopic origin of de Sitter entropy*, arXiv:1801.03012 [INSPIRE].
- [51] C. Pattison, V. Vennin, H. Assadullahi and D. Wands, *Stochastic inflation beyond slow roll*, *JCAP* **07** (2019) 031 [arXiv:1905.06300] [INSPIRE].
- [52] A.M. Polyakov, *Infrared instability of the de Sitter space*, arXiv:1209.4135 [INSPIRE].
- [53] C. Itzykson and J.M. Drouffe, *Statistical Field Theory*. Cambridge University Press, Cambridge, U.K. (1989).
- [54] V.V. Slezov, *Kinetics of First-Order Phase Transitions*, Wiley-VCH Verlag GmbH & Co. KGaA, Weinheim, Germany (2009).

- [55] M. Aizenman, *Geometric Analysis of ϕ^4 Fields and Ising Models (Parts 1 & 2)*, *Commun. Math. Phys.* **86** (1982) 1 [INSPIRE].
- [56] M. Aizenman, *On the renormalized coupling constant and the susceptibility in ϕ^4 in four-dimensions field theory and the Ising model in four-dimensions*, *Nucl. Phys. B* **225** (1983) 261 [INSPIRE].
- [57] W.H. Zurek, *Pointer Basis of Quantum Apparatus: Into What Mixture Does the Wave Packet Collapse?*, *Phys. Rev. D* **24** (1981) 1516 [INSPIRE].
- [58] W.H. Zurek, *Decoherence, einselection, and the quantum origins of the classical*, *Rev. Mod. Phys.* **75** (2003) 715 [quant-ph/0105127] [INSPIRE].
- [59] N.G. Fytas, V. Martín-Mayor, M. Picco and N. Sourlas, *Specific-heat exponent and modified hyperscaling in the 4D random-field Ising model*, *J. Stat. Mech.* **1703** (2017) 033302 [arXiv:1611.09015] [INSPIRE].
- [60] N.G. Fytas and V. Martín-Mayor, *Universality in the Three-Dimensional Random-Field Ising Model*, *Phys. Rev. Lett.* **110** (2013) 227201 [Erratum *ibid.* **111** (2013) 019903; Erratum *ibid.* **111** (2013) 119903].
- [61] N.G. Fytas, V. Martín-Mayor, M. Picco and N. Sourlas, *Phase Transitions in Disordered Systems: The Example of the Random-Field Ising Model in Four Dimensions*, *Phys. Rev. Lett.* **116** (2016) 227201.
- [62] N.G. Fytas, V. Martin-Mayor, M. Picco and N. Sourlas, *Restoration of Dimensional Reduction in the Random-Field Ising Model at Five Dimensions*, *Phys. Rev. E* **95** (2017) 042117 [arXiv:1612.06156] [INSPIRE].
- [63] D.S. Fisher, *Scaling and critical slowing down in random-field Ising systems*, *Phys. Rev. Lett.* **56** (1986) 416 [INSPIRE].
- [64] J.C. Angles d'Auriac, M. Preissmann and R. Rammal, *The random field Ising model: algorithmic complexity and phase transition*, *J. Phys. Lett.* **46** (1985) 173.
- [65] A. Goldberg and R. Tarjan, *Solving minimum-cost flow problems by successive approximation*, in *Proceedings of the nineteenth annual ACM conference on Theory of computing — STOC '87*, New York, U.S.A., 1987, pp. 7–18.
- [66] R. Balian, J.M. Drouffe and C. Itzykson, *Gauge Fields on a Lattice. 2. Gauge Invariant Ising Model*, *Phys. Rev. D* **11** (1975) 2098 [INSPIRE].
- [67] M. Creutz, *Phase Diagrams for Coupled Spin Gauge Systems*, *Phys. Rev. D* **21** (1980) 1006 [INSPIRE].
- [68] E. Kehl, H. Satz and B. Walth, *Critical exponents of Z_2 gauge theory in (3+1)-dimensions*, *Nucl. Phys. B* **305** (1988) 324 [INSPIRE].